Information Retrieval

WS 2013 / 2014

Lecture 2, Tuesday October 29th, 2013

(Ranking, tf.idf, BM25, Vector Space Model, Evaluation)

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Overview of this lecture

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- Organizational
  - Your experiences with Ex. Sheet 1 (inverted index)
- How to rank results
  - Basic principle / scores
  - Formulas: tf.idf and BM25
  - Vector Space Model
  - Quality evaluation: precision, recall, …, nDCG@k

Exercise Sheet #2: compare three ranking formulas with respect to their nDCG@5 for query of your choice

Experiences with ES1 (inverted index)

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- Summary / excerpts last checked October 29, 15:00
  - Liked the style of the lecture / exercises
  - Heard stuff about SVN etc. for the 100th time
  - No major problems for most, good exercise for starters
  - Some overhead for setting up the environment
  - Easy to overlook the implementation note in .TIP file
  - The usual complaints about the style checker

    Tabs vs. spaces, how to place the { … }
  - Put slides and exercise sheet in the SVN, too

    Not so easy as it sounds, but I try to find a way …
  - "Unfortunately couldn't finish, but it was fun."

Ranking 1/4

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- Motivation
  - Queries often return many hits
  - Typically more than one wants to (or even can) look at

    For web search: often millions of documents

    But even for less hits, a proper ranking is **key** to usability, recall the Broccoli demo from Lecture 1
  - So we want to have the most "relevant" hits first
  - Problem: how to measure what is how "relevant"

Ranking 2/4

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- Basic Idea
  - In the inverted lists, for each **doc id** also have a **score**

    uni **17 0.5, 53 0.2, 97 0.3, 127 0.8**

    freiburg **23 0.1, 34 0.8, 53 0.1, 127 0.7**
  - When intersecting lists aggregate (here: **add**) the scores

    uni freiburg **53 0.3, 127 1.5**
  - Then sort the result by score

    uni freiburg **127 1.5, 53 0.3**
  - The entries in the list are referred to as **postings**

    Above, it's only doc id and score, but a **posting** can also contain more information, e.g. the position of a word

Ranking 3/4

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- Generalization
  - We can do the same thing with computing the **union**

    uni 17 0.5 , 53 0.2 , 97 0.3 , 127 0.8

    freiburg 23 0.1 , 34 0.8 , 53 0.1 , 127 0.7

    UNION 17 0.5 , 23 0.1 , 34 0.8 , 53 0.3 , 97 0.3 , 127 1.5

    SORTED 127 1.5 , 34 0.8 , 17 0.5 , 53 0.3 , 97 0.3 , 23 0.1
  - Note: documents which contain only some (or one) of the words can be ranked before documents containing all of the words

    provided the individual scores are high enough
  - This is also called **and-ish** retrieval … like AND, but not exactly

    For ES2 you can continue to use intersection

Ranking 4/4

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- Getting the top-k results
  - A full sort takes time $\Theta(n \cdot \log n)$, where n = #documents
  - Typically only the top-k hits need to be displayed
  - Then a **partial sort** is sufficient: get the k largest elements, for a given k

    This can be computed in time $\Theta(n + k \cdot \log k)$

    k rounds of HeapSort yield time $\Theta(n + k \cdot \log n)$
  - For constant k these are both $\Theta(n)$
  - In C++ there is std::sort and std::partial_sort
  - In Java there is Collections.sort but no partial sort method

For ES2, you can but don't have to use partial sort

Scores 1/8

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- How to compute meaningful scores
  - Let S_1, S_2, S_3, … be the score sums of the documents D_1, D_2, D_3, … for a given keyword query Q
  - **GOAL:** S_i should reflect the relevance of D_i for Q

    in particular: $S_i > S_j$ → D_i more relevant for Q than D_j
  - Obviously a very hard problem

    In particular, it is often less than clear what is the search request behind a given query

    For example: freiburg doctor
  - But it has to be done anyway !

Scores 2/8

- One important factor: tf = term frequency

  tf of a word w in a doc D = how often w occurs in D

  – Problem with mere tf scores: some words are frequent in many documents, regardless of content

university	… ,	**57**	**5**	, … … … ,	**123**	**2**	, …
of	… ,	**57**	**14**	, … … … ,	**123**	**23**	, …
freiburg	… ,	**57**	**3**	, … … … ,	**123**	**1**	, …
SCORE SUM	… ,	**57**	**22**	, … … … ,	**123**	**26**	, …

But the **tf score** for "of" should not count that much for relevance

Scores 3/8

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- Another important factor: **df** = document frequency

  df of a word w = the number of docs containing w

  – For example … for simplicity, numbers will be powers of 2

  $\mathbf{df}_{university}$ = 16.384 , $\mathbf{df}_{of}$ = 524.288 , $\mathbf{df}_{freiburg}$ = 1.024

  – Intuitively, words with a large df should not count as much; thus consider the inverse document frequency

  idf = $\log_2$ (N / df) N = total number of documents

  – For the example df scores above and N = 1.048.576 = 2^{20}

  $\mathbf{idf}_{university}$ = 6 , $\mathbf{idf}_{of}$ = 1, $\mathbf{idf}_{freiburg}$ = 10

  Understand: without the $\mathbf{log_2}$, small differences in **df** would have too much of an effect ; why exactly $\mathbf{log_2}$ → later slide

Scores 4/8

- Combining the two: **tf.idf** = tf · idf = tf · $\log_2$ (N / df)
  - Reconsider our earlier **tf** only example

university	… , **57 5** , … … … , **123 2** , …
of	… , **57 14** , … … … , **123 23** , …
freiburg	… , **57 3** , … … … , **123 1** , …
SCORE SUM	… , **57 22** , … … … , **123 26** , …

  - Now combined with **idf** scores from previous slide

university	… , **57 30** , … … … , **123 12** , …
of	… , **57 14** , … … … , **123 23** , …
freiburg	… , **57 30** , … … … , **123 10** , …
SCORE SUM	… , **57 74** , … … … , **123 45** , …

Scores 5/8

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- Problems with tf.idf in practice
  - The idf part is fine, but the tf part has several problems:
  - Let w be a word, and D_1 and D_2 be two documents
  - **Problem 1** (example)

    If D_1 is longer than D_2, it will tend to have a higher tf for w already because it's longer, not because it's more "about" w
  - **Problem 2** (example)

    If D_1 and D_2 have the same length, and the tf of w in D_1 is twice the tf of w in D_2

    … then it is reasonable to assume that D_1 is more "about" w than D_2, but just a little more, and not twice more

Scores 6/8

$k=0,\ b=0 : tf^* = tf / tf = 1$

$k=\infty,\ b=0 : tf^* = \lim_{k\to\infty} \frac{tf\cdot(k+1)}{k+tf} = \lim_{k\to\infty} \frac{tf(1+\frac{1}{k})}{1+\frac{tf}{k}} = tf$

- **BM25** = Best Match 25, Okapi = an IR system
  - This tf.idf style formula has consistently outperformed other formulas in standard benchmarks over the years

    BM25 score = $tf^* \cdot \log_2 (N / df)$, where

    $tf^* = tf \cdot (k + 1) / (k \cdot (1 - b + b \cdot DL / AVDL) + tf)$

    tf = term frequency, DL = document length, AVDL = average document length

    Standard setting for **BM25**: k = 1.75 and b = 0.75

    Binary: k = 0, b = 0; Normal tf.idf: k = ∞, b = 0
  - There is "theory" behind this formula ... see references
  - Next slide: simple reason why the formula makes sense

For ES2, you can just take #Bytes

Scores 7/8

- Why BM25 makes sense
  - Start with the simple formula tf · idf
  - Replace tf by tf* = tf · (k + 1) / (k + tf)
    - tf* = 0 if and only if tf = 0
    - tf* increases as tf increases
    - tf* → k + 1 as tf → infinity
  - Normalize by the length of the document
    - full normalization: alpha = DL / AVDL
    - some normalization: alpha' = (1 – b) + b · DL / AVDL
    - replace tf* by tf* / alpha'

The same argument goes for the $\log_2$ in idf.

BM25 is the simplest formula with these properties!

e.g. $\alpha = 2$, $b = 0.5$

$\Rightarrow \alpha' = 0.5 + 0.5 \cdot 2 = 1.5$

for $b = 0 \Rightarrow \alpha' = 1$

Scores 8/8

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- Implementation advice
  - The entries in the inverted lists are now elements of a class Posting, each holding a doc id and a score

    Map<String, Array<Posting>> invertedLists;
  - During parsing, compute only basic tf: when a document contains a word multiple times, simply add 1 to the score
  - After the parsing, the length of each inverted list is exactly the df for that word, which also gives you the idf then

    The final tf.idf / BM25 scores can then be obtained by another pass over each of the inverted lists

    See the TIP files for ES2 linked on the Wiki

Vector Space Model 1/4

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- Basic Idea
  - View documents (and queries) as **vectors** in a vector space
  - Each dimension corresponds to a word from the vocabulary
  - Entries can be according to any of our scoring formulas
  - Here is an example

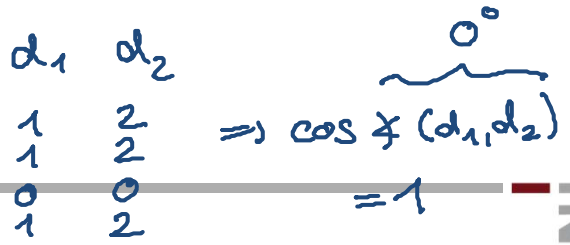
in the example below: tf.idf scores

Document 1: university of freiburg
Document 2: university of karlsruhe
Document 3: freiburg cathedral

QUERY: university freiburg

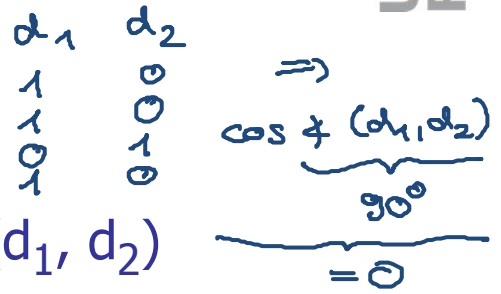
	Doc 1	Doc 2	Doc 3	QUERY
university	1	1	0	1
freiburg	1	0	1	1
karlsruhe	0	1	0	0
of	1	1	0	0
cathedral	0	0	1	0

Vector Space Model 2/4



- Similarity between two documents

  or between a document and a query



  - **Cosine similarity:** $\text{sim}(d_1, d_2) = \cos \text{angle}(d_1, d_2)$

    This is **1** if $d_1 \sim d_2$, and **0** if no word in common

    Advantage: favorable properties for mathematic analysis

  - **Dot product:** d1 • d2 = sum of products of components

    Advantage: easy to compute efficiently ... later slide

  - From linear algebra: $d_1 \bullet d_2 = |d_1| \cdot |d_2| \cdot \cos \text{angle}(d_1, d_2)$

  - Therefore, if the vectors are length normalized ($|\cdot| = 1$) then

    dot product = cosine similarity $\begin{pmatrix}1\\1\\0\\1\end{pmatrix} \bullet \begin{pmatrix}1\\2\\0\\0\end{pmatrix} = 1 \cdot 1 + 1 \cdot 2 + 0 \cdot 0 + 1 \cdot 0 = 3$

Vector Space Model 3/4

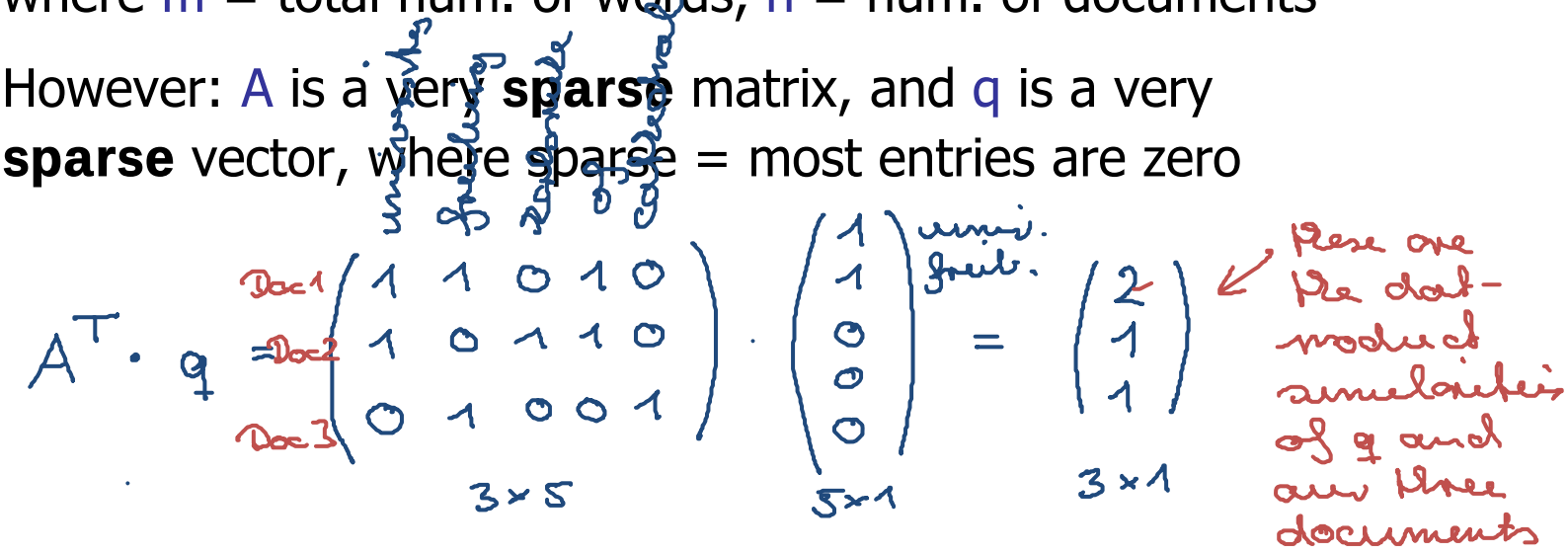
- Computing the dot product similarity

  … of a query to all documents

  – Mathematically, this is just a multiplication of the term-document matrix A with the query q

  – The straightforward algorithm needs time $\Theta(m \cdot n)$, where m = total num. of words, n = num. of documents

  – However: A is a very **sparse** matrix, and q is a very **sparse** vector, where sparse = most entries are zero



Vector Space Model 4/4

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- Computing the dot product similarity
  - Observation 1: Inverted lists are nothing else, but a representation of the non-zero entries of the term-document matrix
  - Observation 2: Computing the dot product of a query Q with **every** document is nothing else but:

    Taking the union of the inverted lists of all words in Q with a non-zero entry and adding up the scores accordingly

Evaluation 1/6

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- How to evaluate the quality of a ranking
  - **Variant 1:** For each query, identify the **ground truth** = all relevant documents for that query

    This is a very time-consuming job, especially for large document collections. But once done, easy + quick re-evaluation after any changes / tuning to your system

    For big data, use services like Amazon's Mechanical Turk
  - **Variant 2:** For each query, manually inspect the result list for relevant documents

    For ES2, just do a manual inspection of the top-5 hits
  - **Variant 3:** In competitions, pooling is sometimes used = manually evaluate only the union of the top-k hits from all participating systems, where e.g. k = 100

Evaluation 2/6

Relevant docs: 17, 45, 51, 107
My search found: 17, 51, 90

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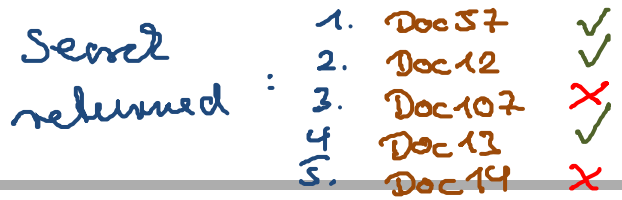
- Precision and Recall, ranking-unaware measures
  - Let tp = the number of relevant docs in the result list (true positives)
  - Let fp = the number of non-relevant docs in the result list (false positives)
  - Let fn = the number of relevant docs missing from the result list (false negatives)
  - Then **precision** is defined as tp / (tp + fp)

    and **recall** is defined as tp / (tp + fn)
  - **F-measure** = harmonic mean of the two

prec. = 2/3 = 67%
rec. = 2/4 = 50%

Evaluation 3/6



- Precision and Recall, ranking-aware measures
  - Precision@k = the precision among the first k docs
  - Precision@R = the precision among the first R docs

    where R is the number of relevant documents
  - Let $k_1 < ... < k_R$ be the ranks of the relevant docs in the result list (rank missing docs randomly or worst-case)

    Average precision = average of $P@k_1, ..., P@k_R$
  - For a set of queries, the **MAP** = mean average precision is the average (over all queries) of the average precisions

Evaluation 4/6

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- Precision-recall curve
  - Average precision is just a single number
  - For a complete picture of the quality of the ranking, plot a precision-recall curve
  - If the x-axis is normalized, these can also be averaged over several queries

Evaluation 5/6

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- More refined measures
  - Sometimes relevance comes in more than one shade, e.g.

    0 = not relevant, 1 = somewhat rel, 2 = very relevant
  - Then a ranking that puts the very relevant docs at the top should be preferred

    Cumulative gain $CG@k = \Sigma_{i=1..k}\ rel_i$

    Discounted CG $DCG@k = rel_1 + \Sigma_{i=2..k}\ rel_i / \log_2 i$
  - **Problem:** CG and DCG are larger for larger result lists
  - **Solution:** normalize by maximally achievable value

    iDCG@k = value of DCG@k for **i**deal ranking

    Normalized DCG $nDCG@k = DCG@k / iDCG@k$

Evaluation 6/6

- Normalized discounted cumulative gain, example

1. very relevant 2
2. relevant 1
3. not relevant 0
4. very relevant 2
5. not relevant 0

$$DGG@5 = 2 + \frac{1}{\log_2 2} + \frac{0}{\log_2 3} + \frac{2}{\log_2 4} + \frac{0}{\log_2 5}$$

$$= 2 + 1 + \frac{2}{2} = 4$$

$$iDGG@5 = 2 + \frac{2}{\log_2 2} + \frac{2}{\log_2 3} + \frac{2}{\log_2 4} + \frac{2}{\log_2 5}$$

References

- In the Raghavan/Manning/Schütze textbook

  Section 6: Scoring, term weighting, vector space model

- Relevant Papers

  The Probabilistic Relevance Framework: BM25 and Beyond

  S. Robertson and H. Zaragoza FnTIR 2009, 333 – 389

- TREC conference (benchmarks)

  http://trec.nist.gov/tracks.html

- Relevant Wikipedia articles

  http://en.wikipedia.org/wiki/Okapi_BM25
  http://en.wikipedia.org/wiki/Precision_and_recall
  http://en.wikipedia.org/wiki/Discounted_cumulative_gain
  http://en.wikipedia.org/wiki/Partial_sorting