

Friday, 4th of March 2022, 14:00-16:15 h, HS 026

## General instructions:

There are five tasks, of which you can select four tasks of your choice. Each task is worth 25 points. If you do all five tasks, we will only count the best four. That is, you can reach a maximum number of 100 points.

You need 50 points to pass the exam. You have 2 hours 15 minutes of time overall. If you do four tasks, this is 30 minutes per task on average plus 15 minutes buffer time.

You are allowed to use one DIN A4 sheet of paper with any contents of your choice related to the lectures and the exercises. You can write on the front and on the back and it can be hand-written or a printout, but it must be from yourself, not from someone else. You are not allowed to use any computing devices or mobile phones, in particular nothing with which you can communicate with others or connect to the Internet or parallel universes.

You may write down your solutions in either English or German.
Please write your solutions on this hand-out, below the description of the tasks! You can also use the back side of the pages. Please write your name and Matrikelnummer on the top of this cover sheet in the framed box. If you need additional pages, please write your name and Matrikelnummer on each of them, too.

## Important:

For the programming tasks: You can use Python, Java, or C++. None of your functions must be longer than TWENTY lines.

For all other tasks: Do not simply write down the final result. It should also be clear how you derived it.

## Good luck!

| T 1 |
| :--- | :--- |
| T 2 |
| T 3 |

Task 1 (Ranking, Evaluation and List Intersection, 25 points)
1.1 (5 points) Consider the following collection of four documents $D_{1}, \ldots, D_{4}$. Write down the term-document matrix for this collection with $t f$.idf scores.
$D_{1}$ : bla bla bla
$D_{2}$ : bli blu blo
$D_{3}$ : bla blo
$D_{4}$ : blu blu
1.2 (5 points) Consider a query "bla bli". Compute the dot-product similarities of the four documents above (using your $t f$.idf scores from 1.1) to that query.
1.3 (5 points) Rank the four documents by the scores from 1.2 (highest score first). Assume that documents $D_{1}$ and $D_{3}$ are relevant. Compute the following metrics: $\mathrm{P} @ 2$, $\mathrm{P} @ \mathrm{R}$, AP (average precision).
1.4 (10 points) Given an array $A$ with integer values in ascending order, we want to locate element $x$ in $A$ with galloping search (an exponential search followed by a binary search). Prove that, if $x$ is contained in $A$, the algorithm needs $O(\log d)$ time, where $d$ is the position of $x$ in $A$.

Task 2 (Zipf's law, Compression, and UTF-8, 25 points)
2.1 (8 points) Zipf's law states the frequency $F$ of the $n$-th most frequent term in a text collection as a function of $n$. State the law and prove that $F$ shows as a straight line with negative slope in a $\log -\log$ plot.
2.2 (7 points) Write the following numbers in Elias-Delta encoding: 1, 7, 16. For each code, underline the part that comes from the Elias-Gamma encoding.
2.3 (5 points) Consider the following encoding: $a=000, b=001, c=01, d=10, e=11$. Find a distribution over $\{a, b, c, d, e\}$ such that this encoding is entropy-optimal, with a proof that it indeed is.
$\mathbf{2 . 4}$ (5 points) Give an example of a two-byte UTF-8 sequence that is a valid encoding of a single Unicode character. Give an example of a two-byte UTF-8 sequence that is an invalid encoding of a single Unicode character. With explanation! Specify each sequence as a bit sequence and as a sequence of four hexadecimal digits.

Task 3 (SPARQL and fuzzy prefix search, 25 points)
3.1 (5 points) Assume we have a knowledge base which includes relations for nationality (relating persons to countries), place_of_birth (relating persons to their place of birth), language_spoken (relating countries to languages), olympic_discipline (relating persons to their olympic disciplines). Express the query Olympians from German-speaking countries with their place of birth and their olympic discipline in SPARQL.
3.2 (10 points) Write a function $\operatorname{ped}(x, y)$ that computes the PED between two strings $x$ and $y$ in $O(|x| \cdot|y|)$ time.
3.3 (10 points) Let $x$ and $y$ be non-empty strings, and let $\delta \in \mathbb{N}$ such that $|x|+\delta<|y|$ and $\operatorname{PED}(x, y) \leq \delta$. Proof that under these conditions $\operatorname{PED}(x, y)=\operatorname{PED}\left(x, y^{\prime}\right)$, where $y^{\prime}$ is the prefix of length $|x|+\delta$ of $y$.

Task 4 (Web Apps and Naive Bayes, 25 Punkte)
4.1 (5 Points) Write a valid HTML page with heading Hardle, three text input fields with ids f1, f2, f3, and a button labeled Guess. The HTML should include a JavaScript file script.js, to be written in Task 4.2.
4.2 (10 Points) Write script.js with the following functionality. Whenever the user presses the Guess button, send the content of input field $f 1$ to a backend at the relative URL/backend. The backend returns a JSON object with 3 integer attributes correct_positions, correct_letters and tries_left. Show the value of correct_positions and correct_letters in input fields $f 2$ and $f 3$. If correct_positions matches the length of the content of field $f 1$, show You win in field $f 1$. If field tries_left is 0 , show You lose. You may use $j$ Query.
4.3 (10 Points) Consider each number from $0 . .15$ as a document with four words, where the words are 0 or 1 and the $i$ th word stands for the $i$ th bit in the binary representation of the number. For example, document 1100 is the number 3 . Each document is labeled with class $A$ if the number is even, and with class $B$ if the number is odd. Train a Naive Bayes classifier on these 16 labeled documents. Determine all probabilities $p_{c}$ and $p_{w c}$ (and write down the intermediate steps). Show that using these probabilities, Naive Bayes predicts class $A$ for the number 1.

Task 5 (Latent Semantic Indexing and Linear Classifiers, 25 points)
5.1 (10 points) Write a function $\operatorname{top}(q, U, S, V, k)$ that returns the index of the top-ranked document for a given query vector $q$, the numpy matrices $U, S, V$ from the singular value decomposition, and the dimension $k$ of the approximation. Use the function transpose for transposition and dot for matrix-matrix or matrix-vector products. In your code, indicate (using a different color) the dimensions of the vectors and matrices and make sure that they match.
5.2 (5 points) Let $H=\left\{x: 2 \cdot x_{1}-2 \cdot x_{2}+x_{3}=7\right\}$ be a 2 D plane in 3D space. Compute the distance of the point $x=(1,2,3)$ to $H$. If you know the formula for the distance computation, you can just use it. Otherwise, derive the formula by writing $x$ as the sum of a point on $H$ and a multiple of the unit normal vector of $H$; the multiple is then the distance.
5.3 (10 points) Let $\sigma(t)=1 /\left(1+e^{-t}\right)$ be the sigmoid function. Prove that $\sigma^{\prime}(t)=\sigma(t) \cdot \sigma(-t)$.

