

Efficient Route Planning

SS 2011

Lecture 11, Friday July 29th, 2011
(Transfer Patterns, course evaluation)

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Overview of this lecture

■ Organizational

- Your results from [Ex. Sheet #7 \(Transit Networks, GTFS\)](#)
- This is the **second to last** lecture

■ Transfer patterns

- A technique for fast routing on transit networks
- We will also discuss why our previous methods have problems for transit networks

■ Exercise sheet ... the last one!

- Fill out the evaluation form for this course → **10 points !**
- Compute the number of transfer patterns between each pair of stations

Feedback from ES#7 (GTFS)

■ Summary / excerpts

- Verständnisprobleme mit der (optionalen) frequencies.txt
- GTFS bothersome to parse
- Implementation advice zum Netzwerk kam etwas spät
- Neuer Rekord: 5-fach verschachtelte Schleife
- Hin- und Rückrichtung sind zwei verschiedene "stations"
- Wichtige Termine standen an. Unser Volk hungert.
- Difficult to find the motivation
- Zeitprobleme jetzt gegen Ende der Vorlesungszeit
- Kein Unterschied zwischen Dijkstra und A*, warum?

Summary of our algorithms so far

- For computing the shortest path from A to B
 - In the following list, Q = average query time, and P = precomputation time for the OSM network of BaWü
 - Times are only the order of magnitude, not exact, hence $\sim$
 - Dijkstra's algorithm ... $Q \sim 0.5 \text{ sec}$, $P \sim 0$
 - A* with the **straightline** heuristic ... $Q \sim 0.2 \text{ sec}$, $P \sim 0$
 - A* with the **landmark** heuristic ... $Q \sim 0.1 \text{ sec}$, $P \sim 10 \text{ sec}$
 - Arc flags ... $Q \sim 1 \text{ msec}$, $P \sim 10 \text{ hours}$
 - Contraction Hierarchies ... $Q \sim 1 \text{ msec}$, $P \sim 1 \text{ min}$
 - Transit Node Routing ... $Q \sim 10 - 100 \text{ } \mu\text{sec}$, $P \sim 1 \text{ min}$
 - Performance on (time-expanded) transit networks?

Dijkstra's algorithms

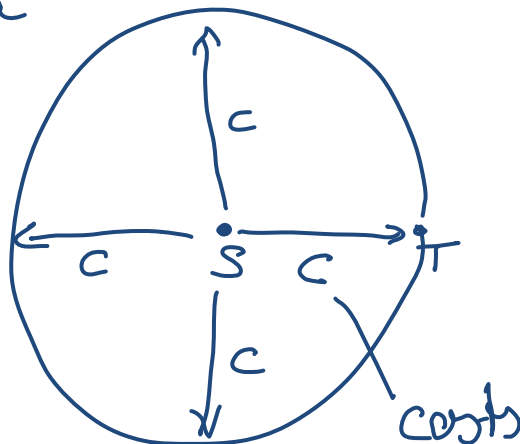
- Performance on time-expanded transit networks
 - One iteration takes $\sim 1 \mu\text{sec} / \text{node}$, as usual
 - But graphs are bigger, for example
 - New York area: $\sim 50\,000$ stations, ~ 2 million nodes
 - When travel time from source to target is T , we settle everything that can be reached within time T , like with the Dijkstra for road networks
 - When T is large or ∞ we search the whole graph ... this happens more often than in road networks, reasons e.g.:
 - bad connectivity by bus / train
 - overnight connections

A* algorithm with straightline heuristic

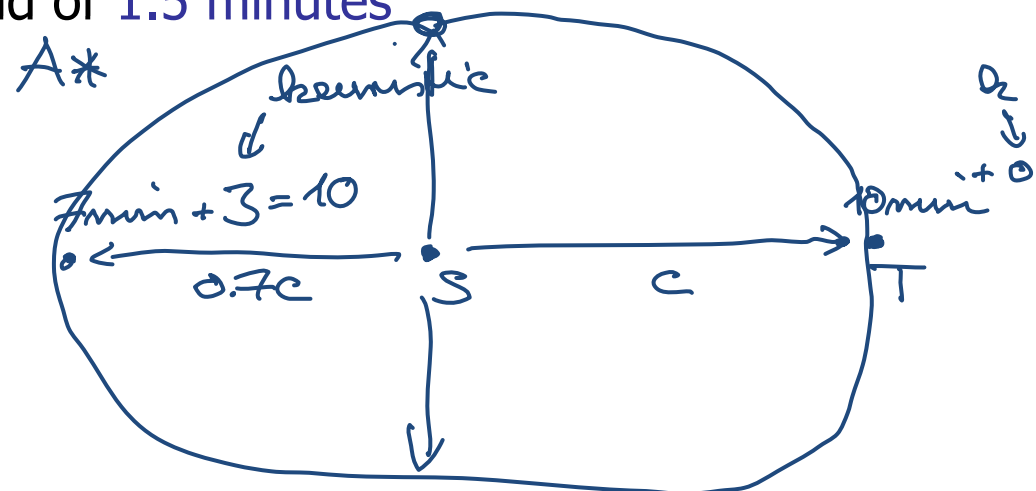
■ Performance on time-expanded transit networks

- Experiment on the Wiki show hardly any improvement over Dijkstra's algorithm, why?
- Consider **Bus 10** from Bärenweg to Siegesdenkmal
- Takes **10 minutes**, straightline distance is ≈ 2.5 km
- Let's say the maximum speed is **100 km/h** (trains!)
- That gives a lower bound of **1.5 minutes**

Dijkstra

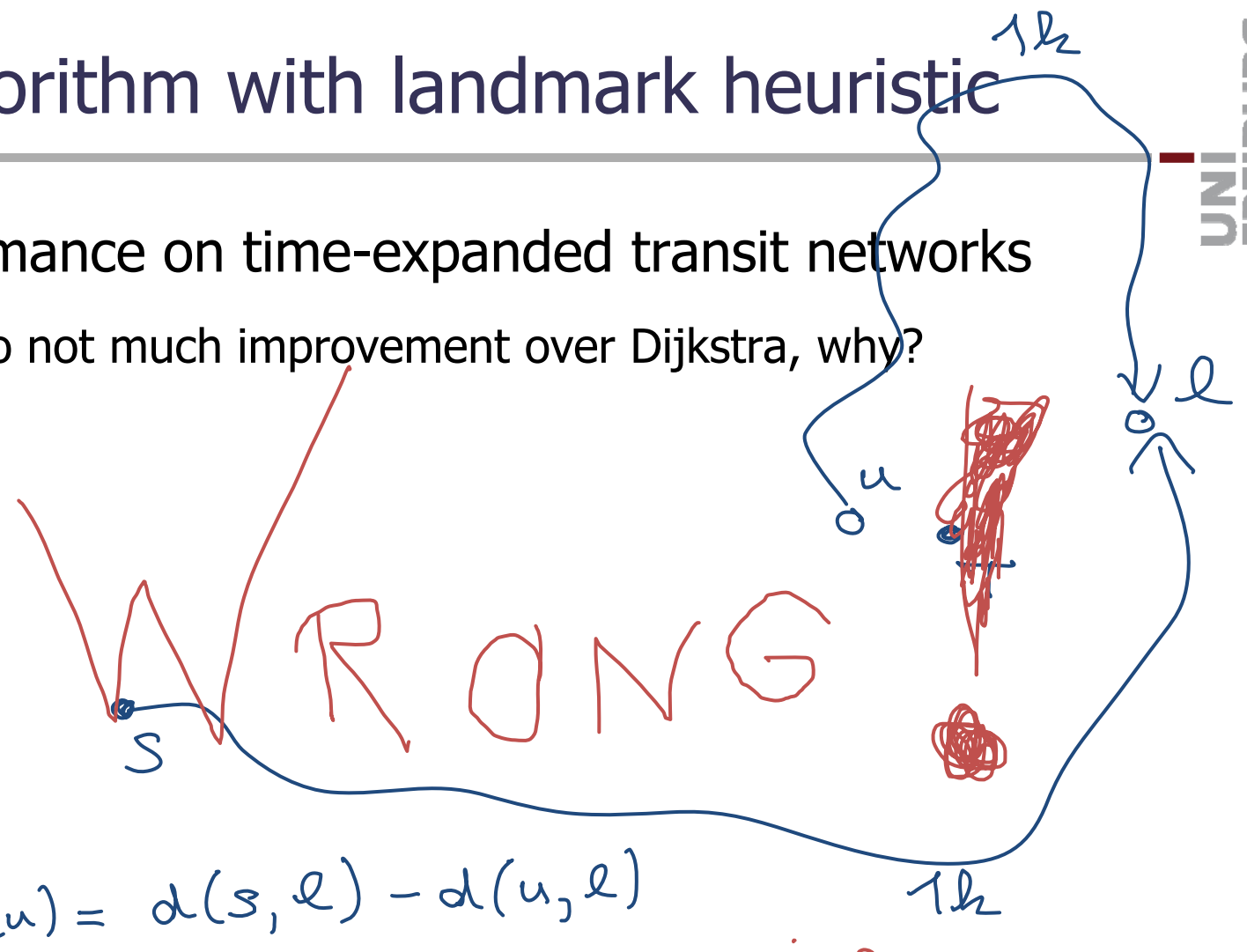


A*



A* algorithm with landmark heuristic

- Performance on time-expanded transit networks
 - Also not much improvement over Dijkstra, why?

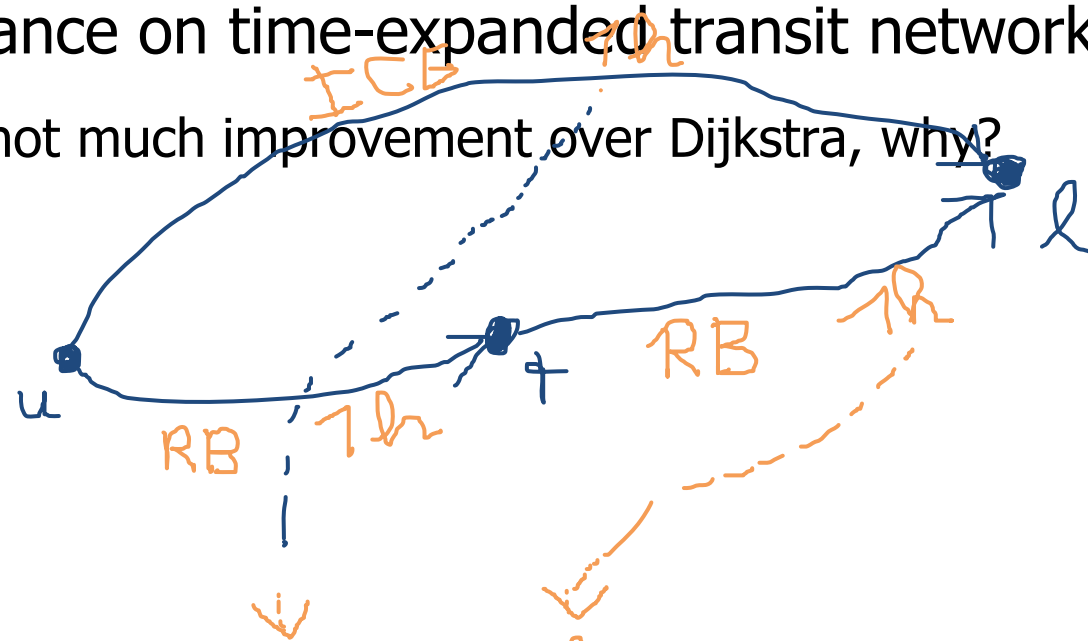

$$g(u) = d(s, l) - d(u, l)$$

Schwacher Zusammenhang zwischen geometrischer Entfernung und Länge (Zeit) eines kürzesten Weges.

A* algorithm with landmark heuristic

■ Performance on time-expanded transit networks

- Also not much improvement over Dijkstra, why?

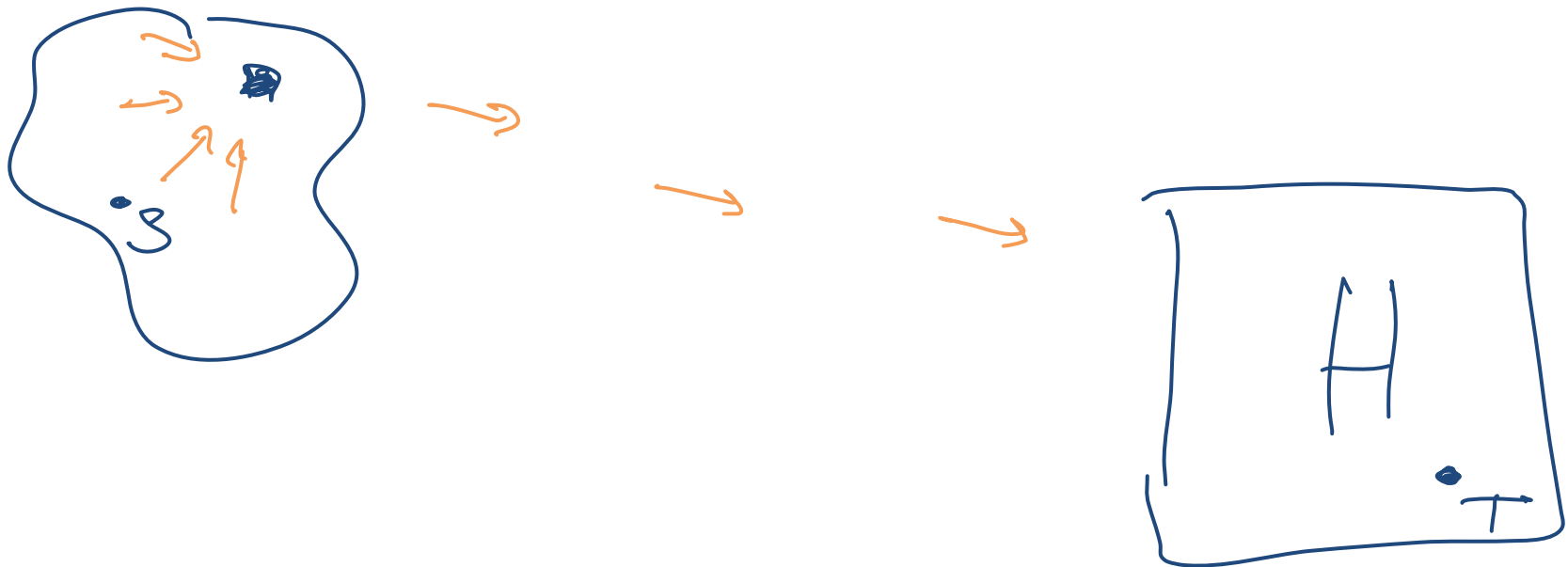


$$0h = 1h - 1h$$
$$g(u) = d(u, l) - d(t, l)$$

Schwacher Zusammenhang zwischen geometrischer Entfernung und Länge (Zeit) eines kürzesten Weges.

Arc flags 1/2

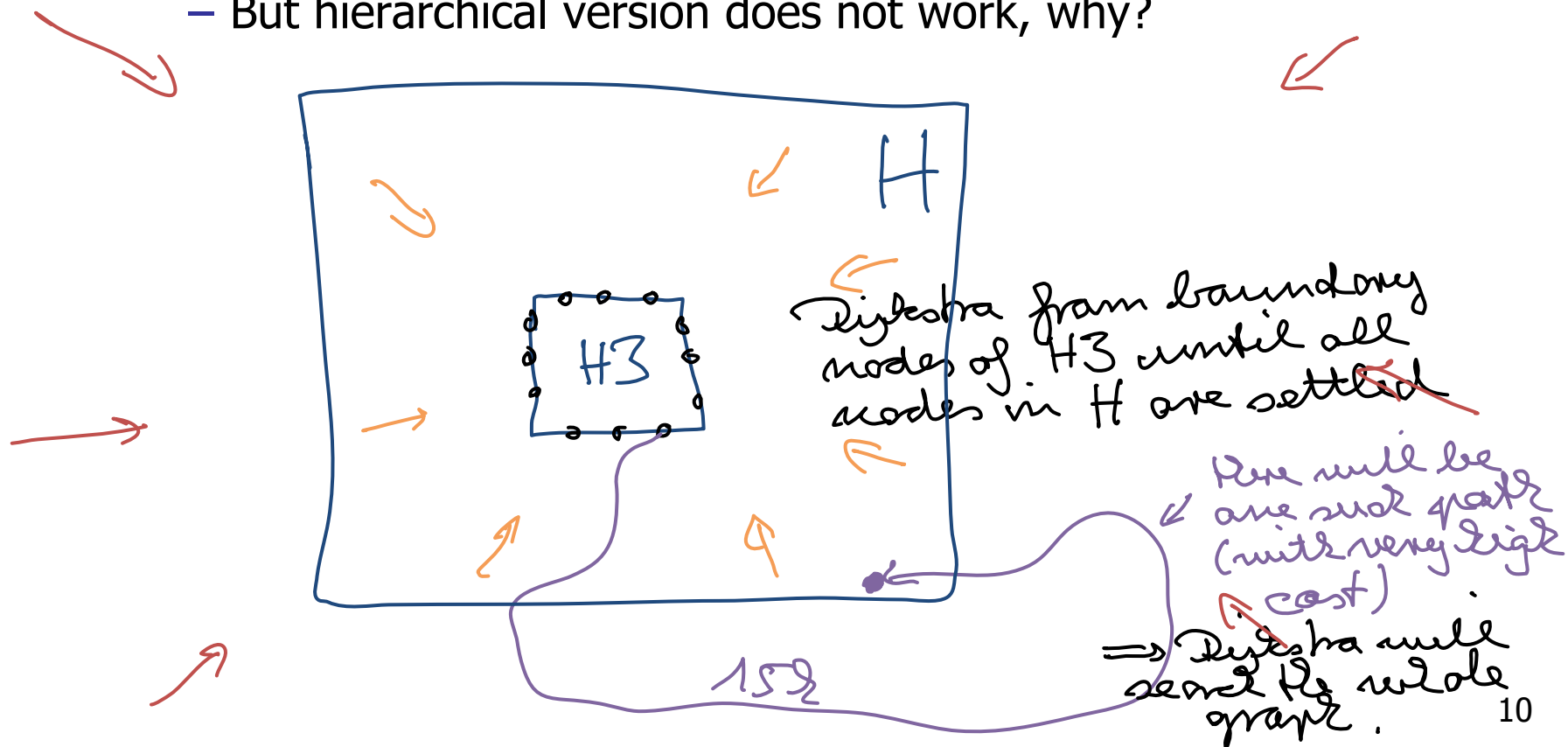
- Performance on time-expanded transit networks
 - Goal direction works well, good query times, why?



Arc flags 2/2

■ Performance on time-expanded transit networks

- But precomputation cost (for the much larger transit graph is enormous) → calls for hierarchical version
- But hierarchical version does not work, why?



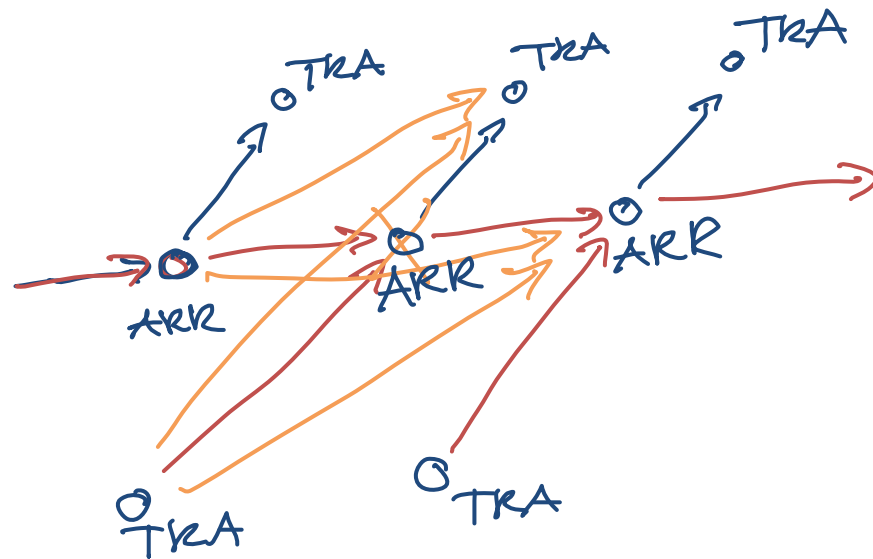
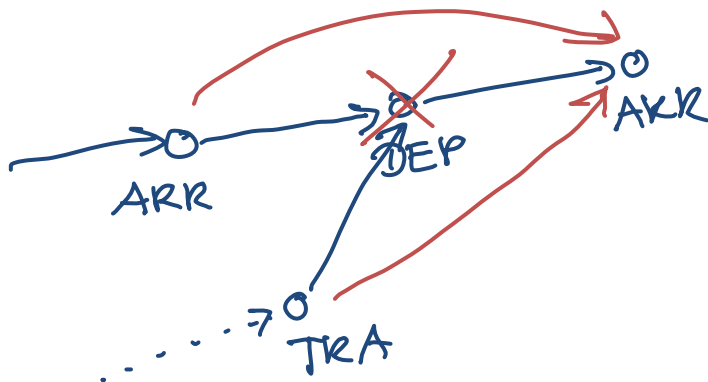
Transit-node routing

- Performance on time-expanded transit networks
 - We can also find small sets of transit / access nodes
 - But how do we compute them efficiently?
 - Also, local queries are not necessarily cheap in transit networks → the "15 hours to the next village problem"

Contraction Hierarchies 1/5

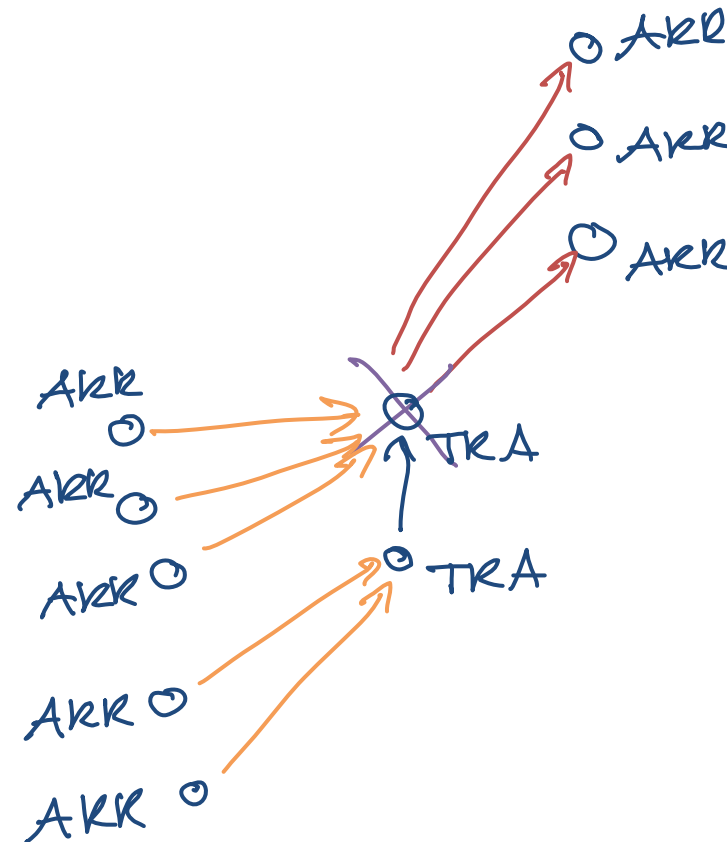
■ Performance on time-expanded transit networks

- Certain nodes can be contracted very well
- We have already seen that the departure nodes can be contracted trivially, and this even saves us arcs
- It seems like arrival nodes can also be contracted without loss



Contraction Hierarchies 2/5

- Performance on time-expanded transit networks
 - When we start to contract transfer nodes, the degree explodes:

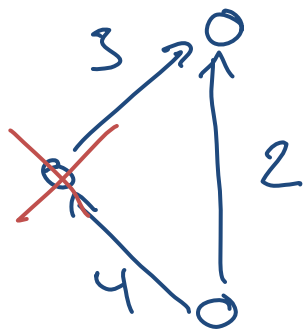


$$3 \times 3 + 2 \times 3 = 15$$

■ Performance on time-expanded transit networks

- Here is one surprising explanation why we need to add so many arcs in transit networks but not in road networks
- Note that in transit networks (with cost = travel time), whenever we contract a node (with in and out degree > 0), we always need to add **all** the potentially necessary shortcuts
- In road networks this is not the case, why this difference?

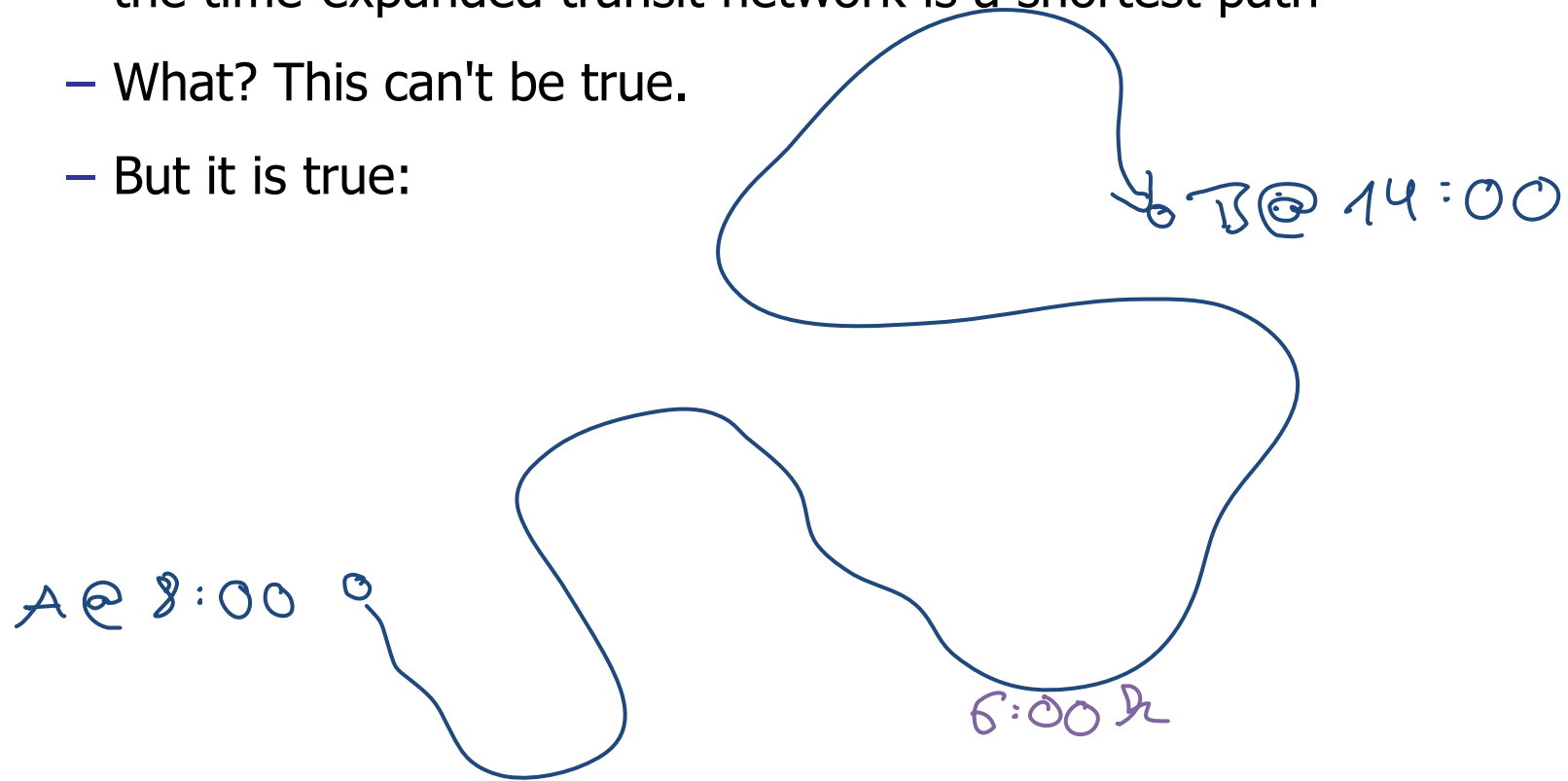
Road network



no need to
add
shortcut

■ Performance on time-expanded transit networks

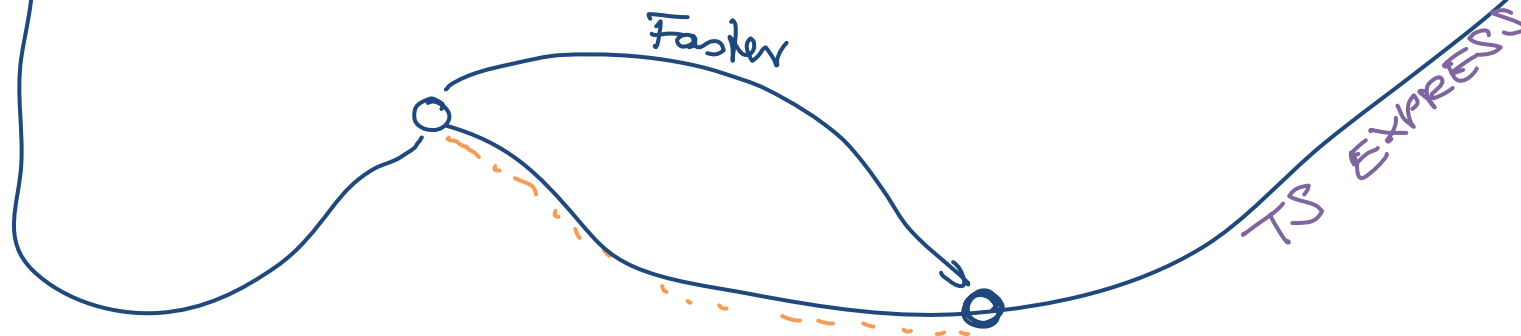
- The reason is that (when cost = travel time), **every** path in the time-expanded transit network is a shortest path
- What? This can't be true.
- But it is true:



Contraction Hierarchies 5/5

■ Performance on time-expanded transit networks

- For multi-criteria costs the situation becomes even worse
- We have seen that in that case, a part P' of a shortest path from A to B might be a non-optimal path itself, and this P' can be arbitrarily far away from A and B
- How are we supposed to figure out that we need P' (when contracting a node on P') with local Dijkstra computations
 - this is at the heart of contraction hierarchies



Summary until here

- None of our algorithms so far
 - ... that is: Dijkstra, A* with the straightline heuristic, A* with the landmark heuristic, arc flags, contraction hierarchies, and transit node routing
 - ... is practical for large transit networks
 - And matters seem to become hopeless when realistic features like transfer buffers and multi-criteria cost function come into play
 - And fully-realistic models pose even more challenges: service days, vehicle restrictions, finding a suitable source and target station, walking between stations, ...

- An algorithm designed for transit networks
 - Trying to exploit what is special about transit networks
 - But what could this be? So far we have only seen things which are harder on transit networks than on road networks
 - Here is one thing special about transit networks:
transfers
 - Even when you take a very long trip, the number of transfers is almost always a very small number
 - And more than that, for a given source and destination, there is only a very limited number of "**patterns**" where it makes sense to transfer

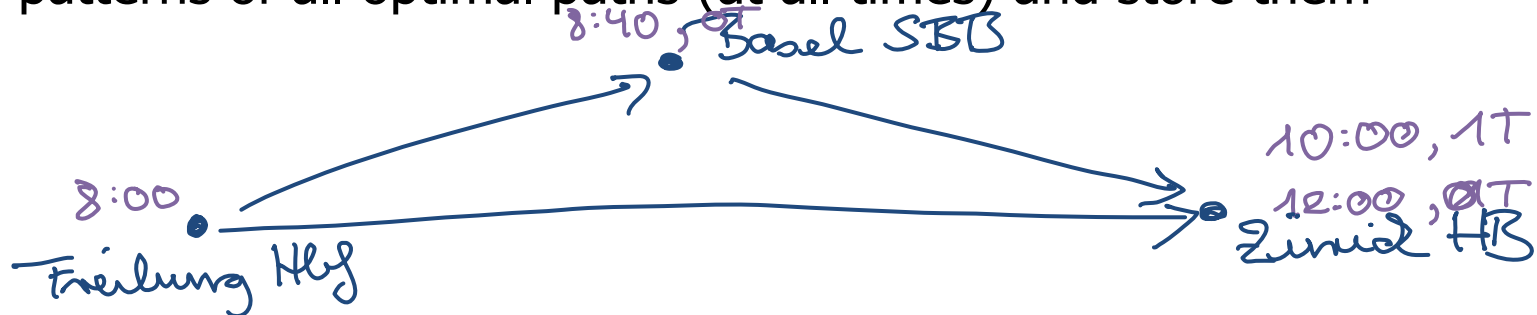
Transfer Patterns 2/3

1. Problem Statement The first problem is to find the maximum value of the function $f(x) = x^2 - 2x + 1$ for $x \in [0, 1]$. The second problem is to find the minimum value of the function $g(x) = x^2 + 2x + 1$ for $x \in [0, 1]$. 2. Solution For the first problem, we can use the derivative method. The derivative of $f(x)$ is $f'(x) = 2x - 2$. Setting $f'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $f(x)$ at $x = 0$ and $x = 1$. We have $f(0) = 1$ and $f(1) = 0$. Therefore, the maximum value of $f(x)$ is 1. For the second problem, we can use the derivative method. The derivative of $g(x)$ is $g'(x) = 2x + 2$. Setting $g'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $g(x)$ at $x = 0$ and $x = 1$. We have $g(0) = 1$ and $g(1) = 4$. Therefore, the minimum value of $g(x)$ is 1.	3. Problem Statement The third problem is to find the maximum value of the function $h(x) = x^3 - 3x^2 + 2x$ for $x \in [0, 2]$. The fourth problem is to find the minimum value of the function $k(x) = x^3 + 3x^2 + 2x$ for $x \in [0, 2]$. 4. Solution For the third problem, we can use the derivative method. The derivative of $h(x)$ is $h'(x) = 3x^2 - 6x + 2$. Setting $h'(x) = 0$, we get $x = 1 \pm \frac{1}{\sqrt{3}}$. Since $x = 1 + \frac{1}{\sqrt{3}}$ is not in the interval $[0, 2]$, we need to check the value of $h(x)$ at $x = 0$, $x = 1 - \frac{1}{\sqrt{3}}$, and $x = 2$. We have $h(0) = 0$, $h(1 - \frac{1}{\sqrt{3}}) = \frac{2}{3\sqrt{3}}$, and $h(2) = 0$. Therefore, the maximum value of $h(x)$ is $\frac{2}{3\sqrt{3}}$. For the fourth problem, we can use the derivative method. The derivative of $k(x)$ is $k'(x) = 3x^2 + 6x + 2$. Setting $k'(x) = 0$, we get $x = -1 \pm \frac{1}{\sqrt{3}}$. Since $x = -1 + \frac{1}{\sqrt{3}}$ is not in the interval $[0, 2]$, we need to check the value of $k(x)$ at $x = 0$ and $x = 2$. We have $k(0) = 0$ and $k(2) = 16$. Therefore, the minimum value of $k(x)$ is 0.	5. Problem Statement The fifth problem is to find the maximum value of the function $m(x) = x^4 - 4x^3 + 6x^2 - 4x + 1$ for $x \in [0, 1]$. The sixth problem is to find the minimum value of the function $n(x) = x^4 + 4x^3 + 6x^2 + 4x + 1$ for $x \in [0, 1]$. 6. Solution For the fifth problem, we can use the derivative method. The derivative of $m(x)$ is $m'(x) = 4x^3 - 12x^2 + 12x - 4$. Setting $m'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $m(x)$ at $x = 0$ and $x = 1$. We have $m(0) = 1$ and $m(1) = 0$. Therefore, the maximum value of $m(x)$ is 1. For the sixth problem, we can use the derivative method. The derivative of $n(x)$ is $n'(x) = 4x^3 + 12x^2 + 12x + 4$. Setting $n'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $n(x)$ at $x = 0$ and $x = 1$. We have $n(0) = 1$ and $n(1) = 16$. Therefore, the minimum value of $n(x)$ is 1.	7. Problem Statement The seventh problem is to find the maximum value of the function $o(x) = x^5 - 5x^4 + 10x^3 - 10x^2 + 5x - 1$ for $x \in [0, 1]$. The eighth problem is to find the minimum value of the function $p(x) = x^5 + 5x^4 + 10x^3 + 10x^2 + 5x + 1$ for $x \in [0, 1]$. 8. Solution For the seventh problem, we can use the derivative method. The derivative of $o(x)$ is $o'(x) = 5x^4 - 20x^3 + 30x^2 - 20x + 5$. Setting $o'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $o(x)$ at $x = 0$ and $x = 1$. We have $o(0) = 1$ and $o(1) = 0$. Therefore, the maximum value of $o(x)$ is 1. For the eighth problem, we can use the derivative method. The derivative of $p(x)$ is $p'(x) = 5x^4 + 20x^3 + 30x^2 + 20x + 5$. Setting $p'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $p(x)$ at $x = 0$ and $x = 1$. We have $p(0) = 1$ and $p(1) = 16$. Therefore, the minimum value of $p(x)$ is 1.
9. Problem Statement The ninth problem is to find the maximum value of the function $q(x) = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$ for $x \in [0, 1]$. The tenth problem is to find the minimum value of the function $r(x) = x^6 + 6x^5 + 15x^4 + 20x^3 + 15x^2 + 6x + 1$ for $x \in [0, 1]$. 10. Solution For the ninth problem, we can use the derivative method. The derivative of $q(x)$ is $q'(x) = 6x^5 - 30x^4 + 60x^3 - 60x^2 + 30x - 6$. Setting $q'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $q(x)$ at $x = 0$ and $x = 1$. We have $q(0) = 1$ and $q(1) = 0$. Therefore, the maximum value of $q(x)$ is 1. For the tenth problem, we can use the derivative method. The derivative of $r(x)$ is $r'(x) = 6x^5 + 30x^4 + 60x^3 + 60x^2 + 30x + 6$. Setting $r'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $r(x)$ at $x = 0$ and $x = 1$. We have $r(0) = 1$ and $r(1) = 16$. Therefore, the minimum value of $r(x)$ is 1.	11. Problem Statement The eleventh problem is to find the maximum value of the function $s(x) = x^7 - 7x^6 + 21x^5 - 35x^4 + 35x^3 - 21x^2 + 7x - 1$ for $x \in [0, 1]$. The twelfth problem is to find the minimum value of the function $t(x) = x^7 + 7x^6 + 21x^5 + 35x^4 + 35x^3 + 21x^2 + 7x + 1$ for $x \in [0, 1]$. 12. Solution For the eleventh problem, we can use the derivative method. The derivative of $s(x)$ is $s'(x) = 7x^6 - 42x^5 + 105x^4 - 140x^3 + 105x^2 - 42x + 7$. Setting $s'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $s(x)$ at $x = 0$ and $x = 1$. We have $s(0) = 1$ and $s(1) = 0$. Therefore, the maximum value of $s(x)$ is 1. For the twelfth problem, we can use the derivative method. The derivative of $t(x)$ is $t'(x) = 7x^6 + 42x^5 + 105x^4 + 140x^3 + 105x^2 + 42x + 7$. Setting $t'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $t(x)$ at $x = 0$ and $x = 1$. We have $t(0) = 1$ and $t(1) = 16$. Therefore, the minimum value of $t(x)$ is 1.	13. Problem Statement The thirteenth problem is to find the maximum value of the function $u(x) = x^8 - 8x^7 + 28x^6 - 56x^5 + 56x^4 - 28x^3 + 8x^2 - 1$ for $x \in [0, 1]$. The fourteenth problem is to find the minimum value of the function $v(x) = x^8 + 8x^7 + 28x^6 + 56x^5 + 56x^4 + 28x^3 + 8x^2 + 1$ for $x \in [0, 1]$. 14. Solution For the thirteenth problem, we can use the derivative method. The derivative of $u(x)$ is $u'(x) = 8x^7 - 56x^6 + 168x^5 - 280x^4 + 224x^3 - 84x^2 + 16x - 8$. Setting $u'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $u(x)$ at $x = 0$ and $x = 1$. We have $u(0) = 1$ and $u(1) = 0$. Therefore, the maximum value of $u(x)$ is 1. For the fourteenth problem, we can use the derivative method. The derivative of $v(x)$ is $v'(x) = 8x^7 + 56x^6 + 168x^5 + 280x^4 + 224x^3 + 84x^2 + 16x + 8$. Setting $v'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $v(x)$ at $x = 0$ and $x = 1$. We have $v(0) = 1$ and $v(1) = 16$. Therefore, the minimum value of $v(x)$ is 1.	15. Problem Statement The fifteenth problem is to find the maximum value of the function $w(x) = x^9 - 9x^8 + 36x^7 - 84x^6 + 126x^5 - 126x^4 + 84x^3 - 36x^2 + 9x - 1$ for $x \in [0, 1]$. The sixteenth problem is to find the minimum value of the function $y(x) = x^9 + 9x^8 + 36x^7 + 84x^6 + 126x^5 + 126x^4 + 84x^3 + 36x^2 + 9x + 1$ for $x \in [0, 1]$. 16. Solution For the fifteenth problem, we can use the derivative method. The derivative of $w(x)$ is $w'(x) = 9x^8 - 72x^7 + 252x^6 - 504x^5 + 630x^4 - 504x^3 + 252x^2 - 72x + 9$. Setting $w'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $w(x)$ at $x = 0$ and $x = 1$. We have $w(0) = 1$ and $w(1) = 0$. Therefore, the maximum value of $w(x)$ is 1. For the sixteenth problem, we can use the derivative method. The derivative of $y(x)$ is $y'(x) = 9x^8 + 72x^7 + 252x^6 + 504x^5 + 630x^4 + 504x^3 + 252x^2 + 72x + 9$. Setting $y'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $y(x)$ at $x = 0$ and $x = 1$. We have $y(0) = 1$ and $y(1) = 16$. Therefore, the minimum value of $y(x)$ is 1.
17. Problem Statement The seventeenth problem is to find the maximum value of the function $z(x) = x^{10} - 10x^9 + 45x^8 - 120x^7 + 210x^6 - 252x^5 + 210x^4 - 120x^3 + 45x^2 - 10x + 1$ for $x \in [0, 1]$. The eighteenth problem is to find the minimum value of the function $aa(x) = x^{10} + 10x^9 + 45x^8 + 120x^7 + 210x^6 + 252x^5 + 210x^4 + 120x^3 + 45x^2 + 10x + 1$ for $x \in [0, 1]$. 18. Solution For the seventeenth problem, we can use the derivative method. The derivative of $z(x)$ is $z'(x) = 10x^9 - 90x^8 + 360x^7 - 840x^6 + 1260x^5 - 1260x^4 + 840x^3 - 360x^2 + 90x - 10$. Setting $z'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $z(x)$ at $x = 0$ and $x = 1$. We have $z(0) = 1$ and $z(1) = 0$. Therefore, the maximum value of $z(x)$ is 1. For the eighteenth problem, we can use the derivative method. The derivative of $aa(x)$ is $aa'(x) = 10x^9 + 90x^8 + 360x^7 + 840x^6 + 1260x^5 + 1260x^4 + 840x^3 + 360x^2 + 90x + 10$. Setting $aa'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $aa(x)$ at $x = 0$ and $x = 1$. We have $aa(0) = 1$ and $aa(1) = 16$. Therefore, the minimum value of $aa(x)$ is 1.	19. Problem Statement The nineteenth problem is to find the maximum value of the function $bb(x) = x^{11} - 11x^{10} + 55x^9 - 165x^8 + 330x^7 - 462x^6 + 462x^5 - 330x^4 + 165x^3 - 55x^2 + 11x - 1$ for $x \in [0, 1]$. The twentieth problem is to find the minimum value of the function $cc(x) = x^{11} + 11x^{10} + 55x^9 + 165x^8 + 330x^7 + 462x^6 + 462x^5 + 330x^4 + 165x^3 + 55x^2 + 11x + 1$ for $x \in [0, 1]$. 20. Solution For the nineteenth problem, we can use the derivative method. The derivative of $bb(x)$ is $bb'(x) = 11x^{10} - 110x^9 + 495x^8 - 1320x^7 + 2310x^6 - 2772x^5 + 2310x^4 - 1320x^3 + 495x^2 - 110x + 11$. Setting $bb'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $bb(x)$ at $x = 0$ and $x = 1$. We have $bb(0) = 1$ and $bb(1) = 0$. Therefore, the maximum value of $bb(x)$ is 1. For the twentieth problem, we can use the derivative method. The derivative of $cc(x)$ is $cc'(x) = 11x^{10} + 110x^9 + 495x^8 + 1320x^7 + 2310x^6 + 2772x^5 + 2310x^4 + 1320x^3 + 495x^2 + 110x + 11$. Setting $cc'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $cc(x)$ at $x = 0$ and $x = 1$. We have $cc(0) = 1$ and $cc(1) = 16$. Therefore, the minimum value of $cc(x)$ is 1.	21. Problem Statement The twenty-first problem is to find the maximum value of the function $dd(x) = x^{12} - 12x^{11} + 66x^{10} - 220x^9 + 495x^8 - 924x^7 + 1386x^6 - 1386x^5 + 924x^4 - 495x^3 + 220x^2 - 66x + 12x - 1$ for $x \in [0, 1]$. The twenty-second problem is to find the minimum value of the function $ee(x) = x^{12} + 12x^{11} + 66x^{10} + 220x^9 + 495x^8 + 924x^7 + 1386x^6 + 1386x^5 + 924x^4 + 495x^3 + 220x^2 + 66x + 12x + 1$ for $x \in [0, 1]$. 22. Solution For the twenty-first problem, we can use the derivative method. The derivative of $dd(x)$ is $dd'(x) = 12x^{11} - 132x^{10} + 660x^9 - 1980x^8 + 3960x^7 - 6468x^6 + 8712x^5 - 8712x^4 + 6468x^3 - 3960x^2 + 1980x - 660x + 12$. Setting $dd'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $dd(x)$ at $x = 0$ and $x = 1$. We have $dd(0) = 1$ and $dd(1) = 0$. Therefore, the maximum value of $dd(x)$ is 1. For the twenty-second problem, we can use the derivative method. The derivative of $ee(x)$ is $ee'(x) = 12x^{11} + 132x^{10} + 660x^9 + 1980x^8 + 3960x^7 + 6468x^6 + 8712x^5 + 8712x^4 + 6468x^3 + 3960x^2 + 1980x + 660x + 12$. Setting $ee'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $ee(x)$ at $x = 0$ and $x = 1$. We have $ee(0) = 1$ and $ee(1) = 16$. Therefore, the minimum value of $ee(x)$ is 1.	23. Problem Statement The twenty-third problem is to find the maximum value of the function $ff(x) = x^{13} - 13x^{12} + 78x^{11} - 286x^{10} + 715x^9 - 1430x^8 + 2145x^7 - 2431x^6 + 2145x^5 - 1430x^4 + 715x^3 - 286x^2 + 78x - 13x + 1$ for $x \in [0, 1]$. The twenty-fourth problem is to find the minimum value of the function $gg(x) = x^{13} + 13x^{12} + 78x^{11} + 286x^{10} + 715x^9 + 1430x^8 + 2145x^7 + 2431x^6 + 2145x^5 + 1430x^4 + 715x^3 + 286x^2 + 78x + 13x + 1$ for $x \in [0, 1]$. 24. Solution For the twenty-third problem, we can use the derivative method. The derivative of $ff(x)$ is $ff'(x) = 13x^{12} - 156x^{11} + 858x^{10} - 2860x^9 + 6435x^8 - 11440x^7 + 15444x^6 - 15444x^5 + 11440x^4 - 6435x^3 + 2860x^2 - 858x + 156x - 13$. Setting $ff'(x) = 0$, we get $x = 1$. Since $x = 1$ is the right endpoint of the interval $[0, 1]$, we need to check the value of $ff(x)$ at $x = 0$ and $x = 1$. We have $ff(0) = 1$ and $ff(1) = 0$. Therefore, the maximum value of $ff(x)$ is 1. For the twenty-fourth problem, we can use the derivative method. The derivative of $gg(x)$ is $gg'(x) = 13x^{12} + 156x^{11} + 858x^{10} + 2860x^9 + 6435x^8 + 11440x^7 + 15444x^6 + 15444x^5 + 11440x^4 + 6435x^3 + 2860x^2 + 858x + 156x + 13$. Setting $gg'(x) = 0$, we get $x = -1$. Since $x = -1$ is not in the interval $[0, 1]$, we need to check the value of $gg(x)$ at $x = 0$ and $x = 1$. We have $gg(0) = 1$ and $gg(1) = 16$. Therefore, the minimum value of $gg(x)$ is 1.

Transfer Patterns 3/3

■ The basic idea on one slide

- The **transfer pattern** of a path = the sequence of stations on the path where one boards, transfers, or alights
- Idea: for each pair of stations, precompute all transfer patterns of all optimal paths (at all times) and store them



- Then, at query time, do a time-dependent Dijkstra computation on this so-called **query graph**, where each arc evaluation is again a shortest path query, but restricted to **no transfers**
- Such **direct-connection** queries are easy to compute fast

Direct-Connection Queries

- One table per "line", let's call this one **L17**

Stations:	S154	S97	S987	S111	...
Time from start:	0min	7min	12min	21min	...
Start times:	8:15	9:15	10:15	11:20	12:20 ...

- Lines per station (with positions in the respective line table)

Station **S97**: (**L8, 4**) (**L17, 2**) (**L34, 5**) (**L87, 17**) ...

Station **S111**: (**L9, 1**) (**L13, 5**) (**L17, 4**) (**L55, 16**) ...

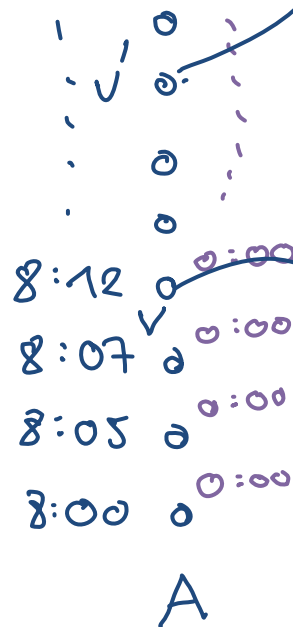
- Example query from **S97 @ 10:20** to **S111**

- Intersect the lists of the two stations : (**L17, 2 → 4**) ...
- Find time from start to **S97** and to **S111** : **7min** and **21min**
- Find first start time after **10:20 – 7min** : **10:15** → **depart 10:22**
- Compute arrival time at **S111** : **10:15 + 21min** → **arrive 10:36**

Transfer patterns precomputation 1/4

■ Can be done via a Set-Dijkstra search

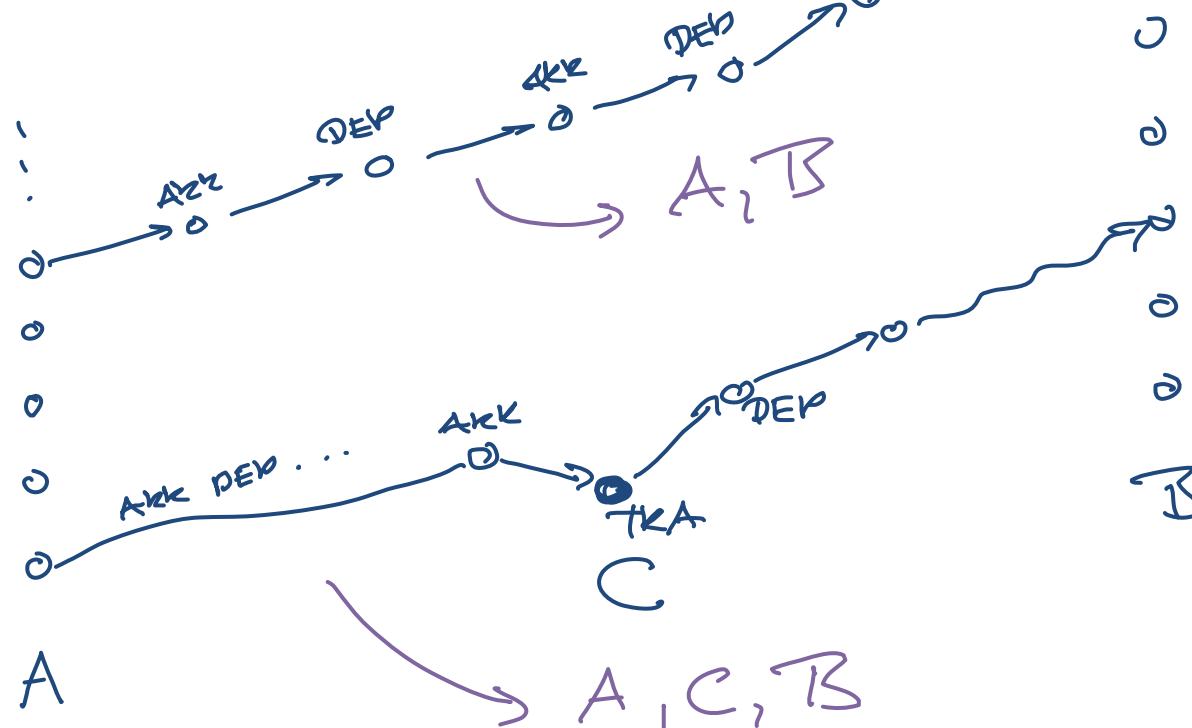
- For each station A , do a Dijkstra starting from **all nodes** at that station (all with cost = travel time zero)
- For each other node u in the graph, this will give us the path from the latest node at A so that u can still be reached



$$d(v, u) = \min \{ d(w, u) : w \in A \}$$

Transfer patterns precomputation 2/4

- Now we have all optimal paths at all times
 - To obtain the transfer patterns for a station pair (A, B) , simply trace back, in the Dijkstra search from A , the paths from all nodes in B and keep track of the transfers



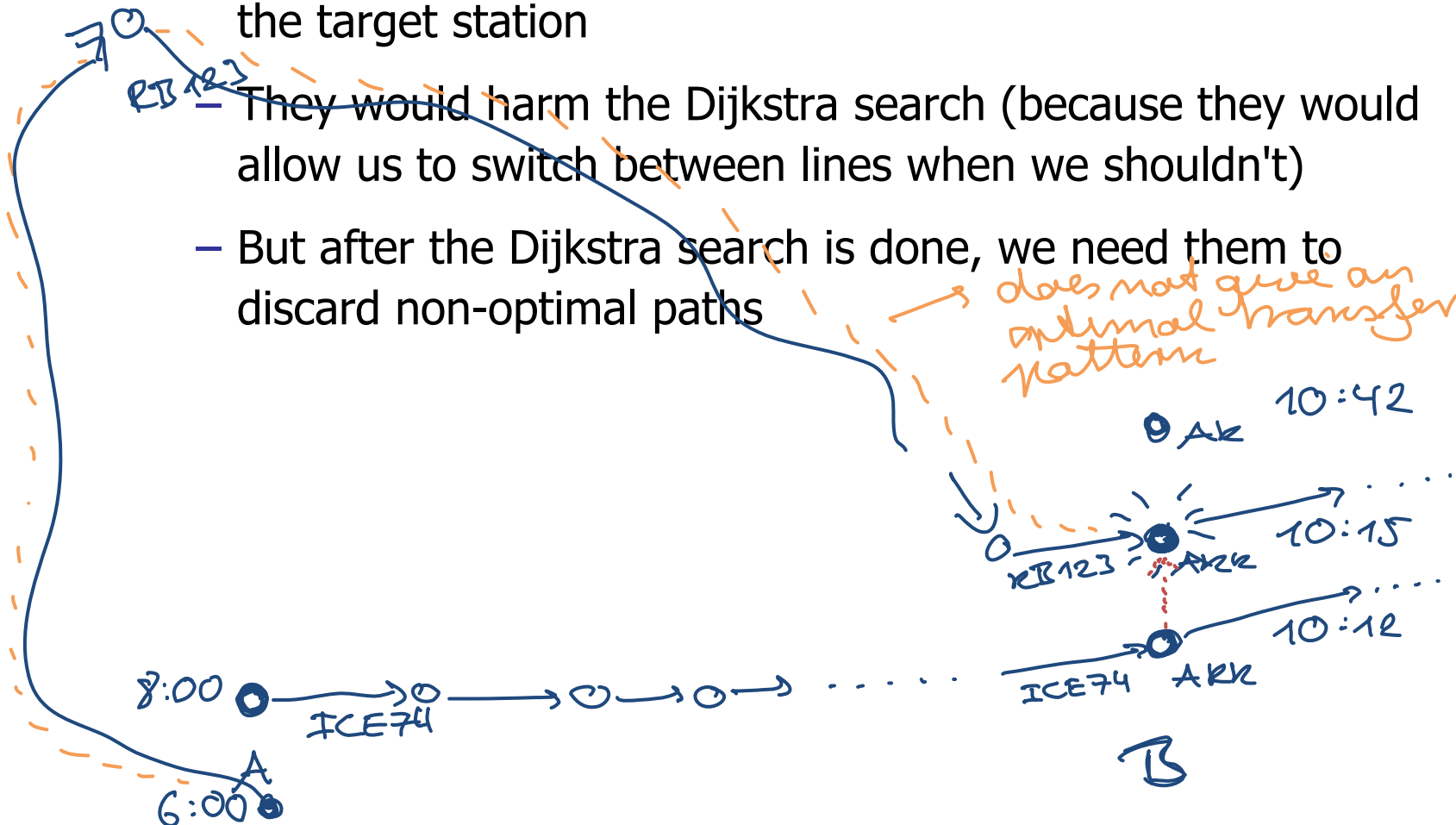
Transfer patterns precomputation 3/4

■ Beware of non-optimal paths to arrival nodes

- Note that there are no arcs between the arrival nodes at the target station

- They would harm the Dijkstra search (because they would allow us to switch between lines when we shouldn't)

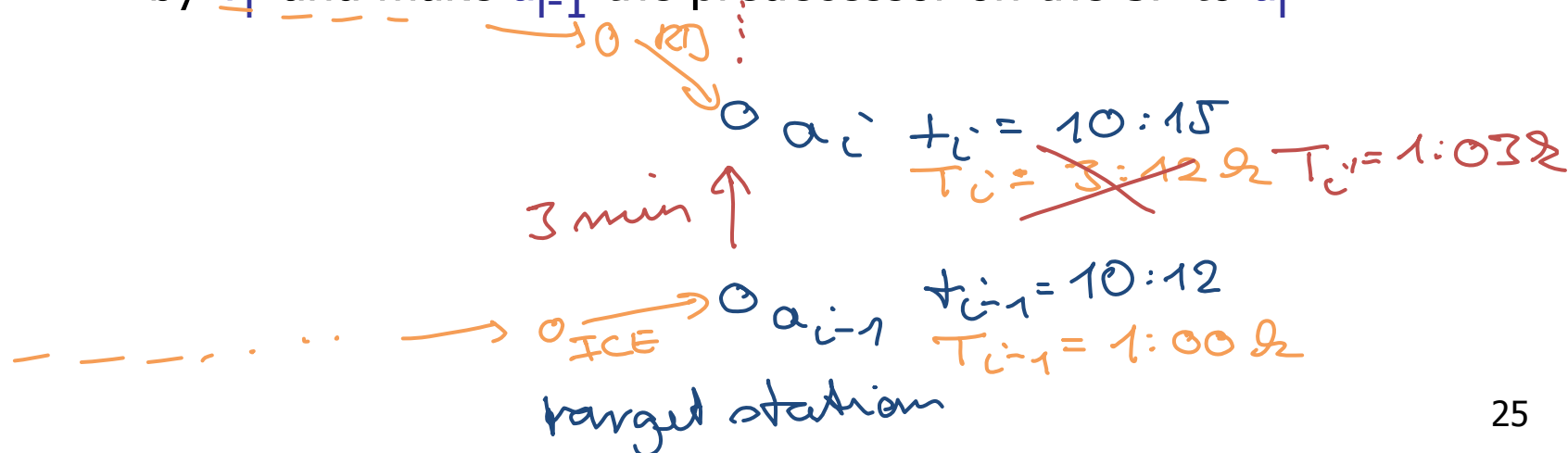
- But after the Dijkstra search is done, we need them to discard non-optimal paths



Transfer patterns precomputation 4/4

■ Arrival-loop algorithm for a target station B

- Order the arrival nodes by time $t_1 \leq t_2 \leq t_3 \leq \dots$ and call the corresponding arrival nodes $a_1, a_2, a_3, \dots$
- Do the following in the order of increasing time
- Let T_{i-1} and T_i be the travel time of the shortest path to a_{i-1} and a_i , respectively
- If $T_i' := T_{i-1} + (t_i - t_{i-1}) \leq T_i$, replace the travel time at a_i by T_i' and make a_{i-1} the predecessor on the SP to a_i



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