Efficient Route Planning

SS 2012

Lecture 2, Wednesday May 2nd, 2012
(Dijkstra's algorithm, Connected Components)

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Overview of this lecture

- Organizational
  - Your feedback and results on Exercise Sheet 1
  - Course Systems: Jenkins
- Dijkstra's Algorithm
  - Idea + Example
  - Correctness proof
  - Implementation advice
  - Connected Components (CCs) using Dijkstra
  - Exercise Sheet 2: Implement Dijkstra + use it to compute the largest CC + use it for some random shortest path queries on Saarland and BaWü

Your Feedback on Exercise Sheet 1

- Summary / excerpts last checked May 2, 15:42
  - Interesting / entertaining, but also quite time-consuming
    - 6 hours for some, up to 15 / 20 / 30 hours for others
    - lack of programming practice
    - setup problems: gtest, SVN, Linux, IDE, etc.
  - Implementation advice from the lecture was useful
  - Memory problems with Java and the BaWü dataset
  - Parsers like Xerces are slow and use a lot of memory
  - Some fights with checkstyle / cpplint
  - How to compute with latitude-longitude coordinates?
  - Let's look at your results ...

Computing with lat-lng coordinates

- Distance in meters between two such coordinates
  - You can use the following approximations
    - one degree of latitude = 111,229 meters
    - one degree of longitude = 71,695 meters
  - Using this, you can easily compute

```
int diffLat = ...;  // Difference of latitude in meters.
int diffLng = ...;  // Difference of longitude in meters.
```

  - From that you can compute the distance in meters

```
int dist = sqrt(diffLat * diffLat + diffLng * diffLng);
```

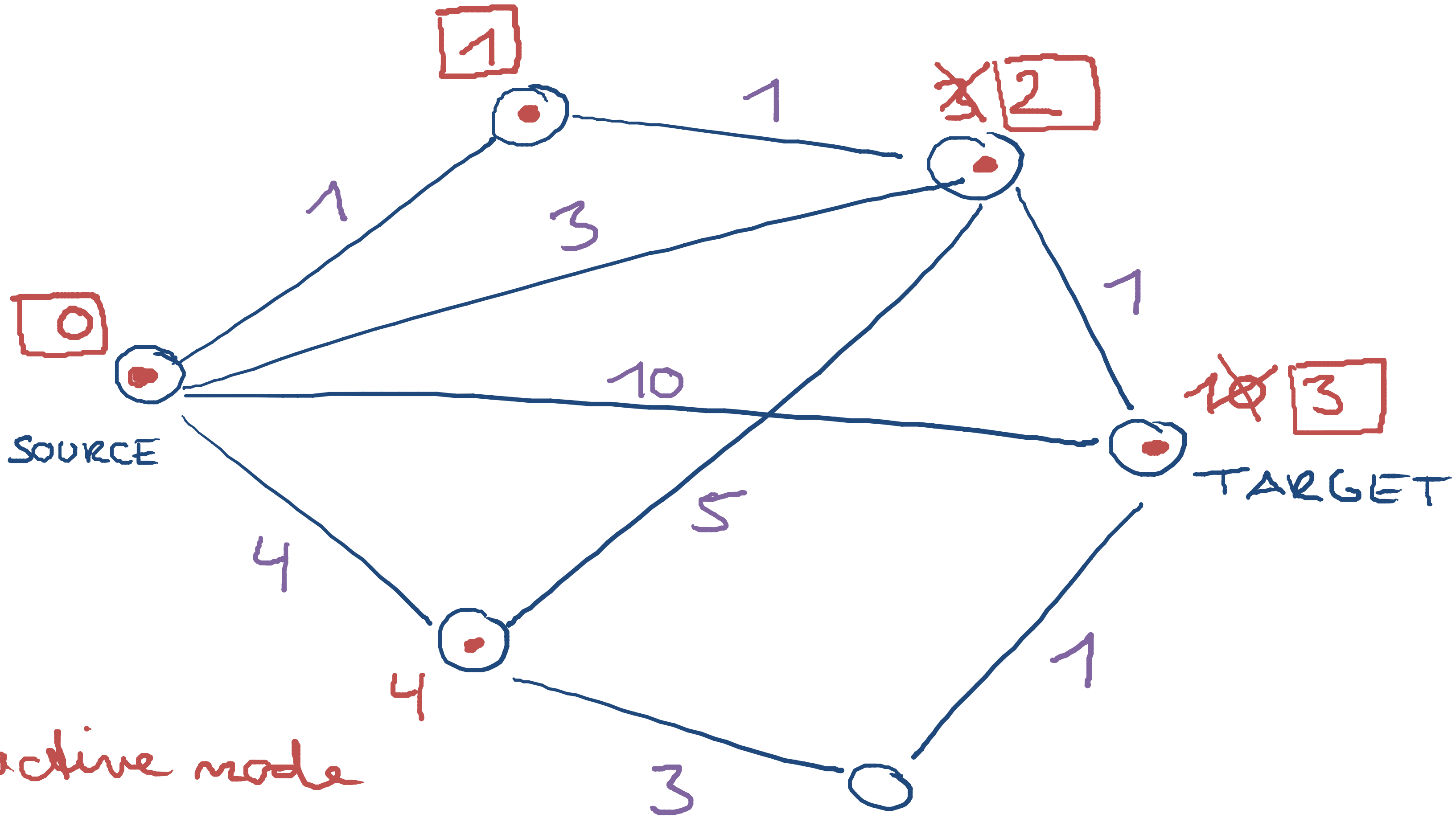
Shortest Path Queries

- Point to point queries
  - For most of this lecture, we are interested in finding the shortest path (path of minimal cost) between two given nodes A and B, called source and target node
  - The cost of a path is simply the sum of the costs of the arcs along the graph
  - The standard algorithm for this task is Dijkstra's algorithm

Dijkstra's algorithm

- You have probably heard it before, here is a recap:
  - Maintains a priority queue of active nodes with tentative distances
  - Initially only the start node is active, with tentative distance 0, all other tentative distances are ∞
  - In each iteration, pick the active node with the **smallest** tentative distance and change its status from active to settled
    - if all arc costs are non-negative, the tentative distance of each settled node is guaranteed to be the correct distance
  - Relax the outgoing arcs = see if the tentative distances of the adjacent nodes can be improved, if yes do so
  - Stop when the target node is settled

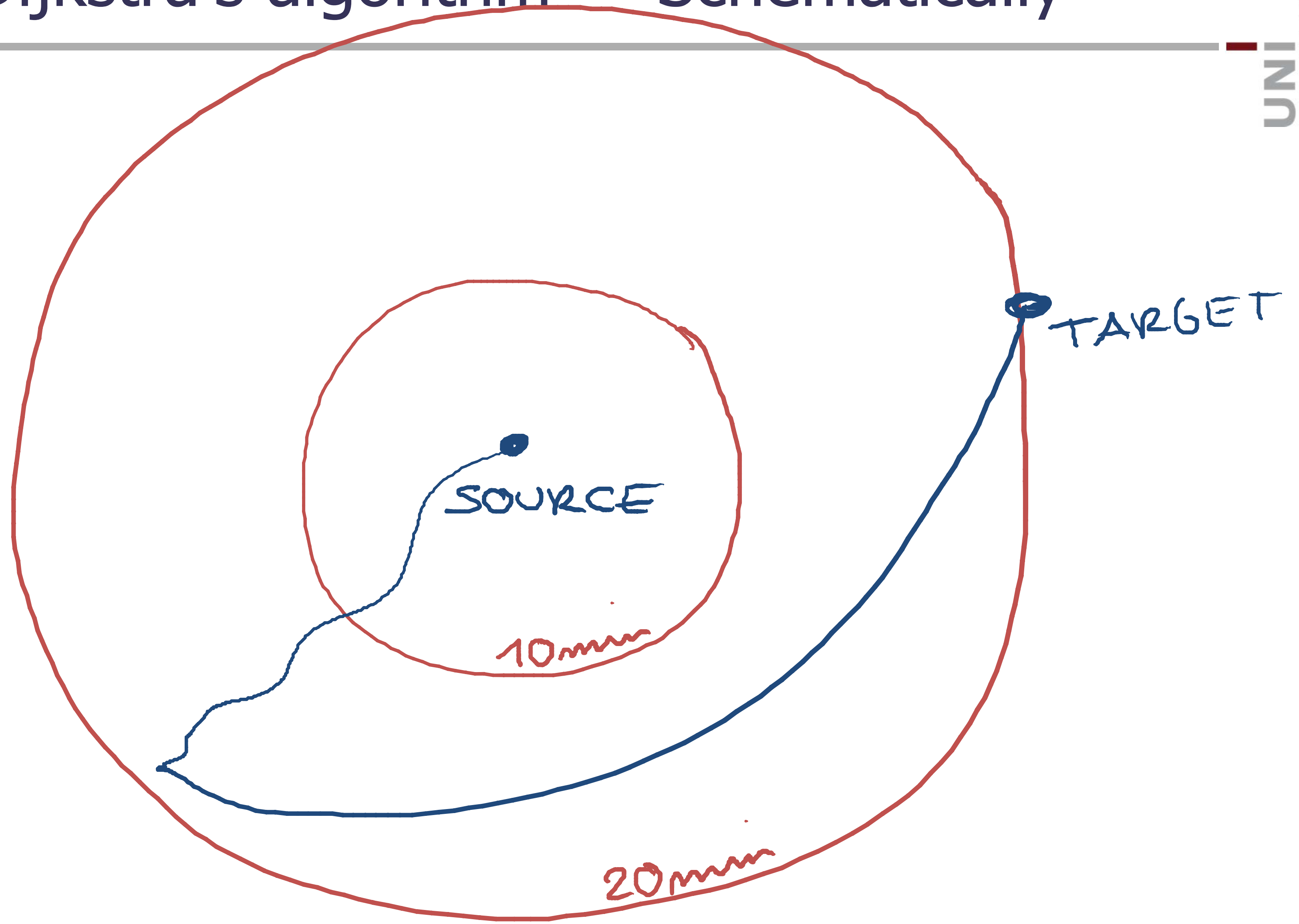
Dijkstra's algorithm — Example



• active node

☐ settled node

Dijkstra's algorithm — Schematically



Dijkstra's algorithm — Properties

- Some basic properties
  - When the target node has been settled, with cost c, than **all** other nodes with cost $< c$ have been settled, too
    - wost case: all nodes reachable from source are settled
  - Running time is $O((m + n) \cdot \log n)$, where
    - m = number of relaxed arcs (worst case: all arcs)
    - n = number of settled nodes (worst case: all nodes)
  - The $\log n$ is the cost of a priority queue (PQ) operation
    - one (potential) insert per arc, one deleteMin per node
    - for a state-of-the-art PQ: 1 µs / deleteMin dominates
    - hence Dijkstra can settle ≈ 1 million nodes / second

Dijkstra — Correctness proof 1/3

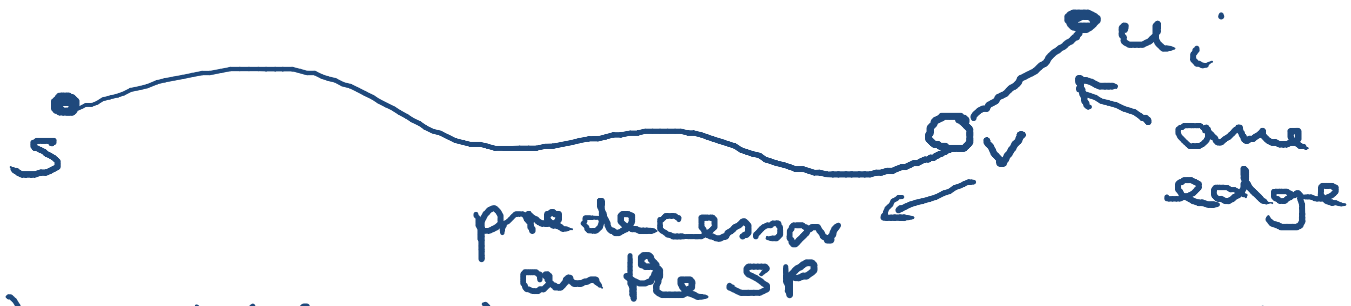
- Let s be our source node
  - Let's first make the simplifying assumptions that the $\mathrm{dist}(s, u)$ are **distinct** for all nodes u
  - Then we can order the nodes $u_1, u_2, u_3, \ldots$

    such that $\mathrm{dist}(s, u_1) < \mathrm{dist}(s, u_2) < \mathrm{dist}(s, u_3) < \ldots$
  - We want to prove that, at the end of the computation,
    - the tentative distance $\mathrm{dist}[u_i]$ for each node u_i satisfies $\mathrm{dist}[u_i] = \mathrm{dist}(s, u_i)$
  - More specifically, we can show that in the i-th iteration
    - Dijkstra's algorithm settles node u_i
    - And at that point $\mathrm{dist}[u_i] = \mathrm{dist}(s, u_i)$

Dijkstra — Correctness proof 2/3

- We show by induction over i
  - that in the i-th iteration, we have $dist[u_j] = dist(s, u_j)$ for all $j \leq i$, and node u_i will be settled in that iteration

Let's look at the SP from s to u_i



$dist(s,v) < dist(s,u_i)$ assuming $c(v,u_i) > 0$

Then, by assumption, v is one of the $u_1, \dots, u_{i-1}$

Let $j < i$: $v = u_j$

Dijkstra — Correctness proof 3/3

By induction hypothesis, v was settled in round $j < i$ and $dist[v] = dist(s, v)$.

But when v was settled, $dist[u_i]$ was set to $dist(s,v) + c(v, u_i) = dist(s, u_i)$.

Let's look at u_j with $j > i$.

For them $dist(u_j) \geq dist(s, u_j) > dist(s, u_i)$

Therefore u_i settled before each such u_j.

$\Longrightarrow$ u_i is settled in round i
with $dist[u_i] = dist(s, u_i)$ ∎

Dijkstra — Implementation advice 1/3

- Where to implement it in your code, two options:
  - As another method in your class RoadNetwork
  - In a separate class DijkstrasAlgorithm
  - I recommend the second option, reasons include:
    - gives you more freedom to extend it later
    - has or will have quite some complexity on its own, and so merits a class on its own
    - each of the more sophisticated algorithms to come will also have a class on their own
    - enables base class for all shortest path algorithms
  - You find a skeleton for the second option on the Wiki

Dijkstra — Implementation advice 2/3

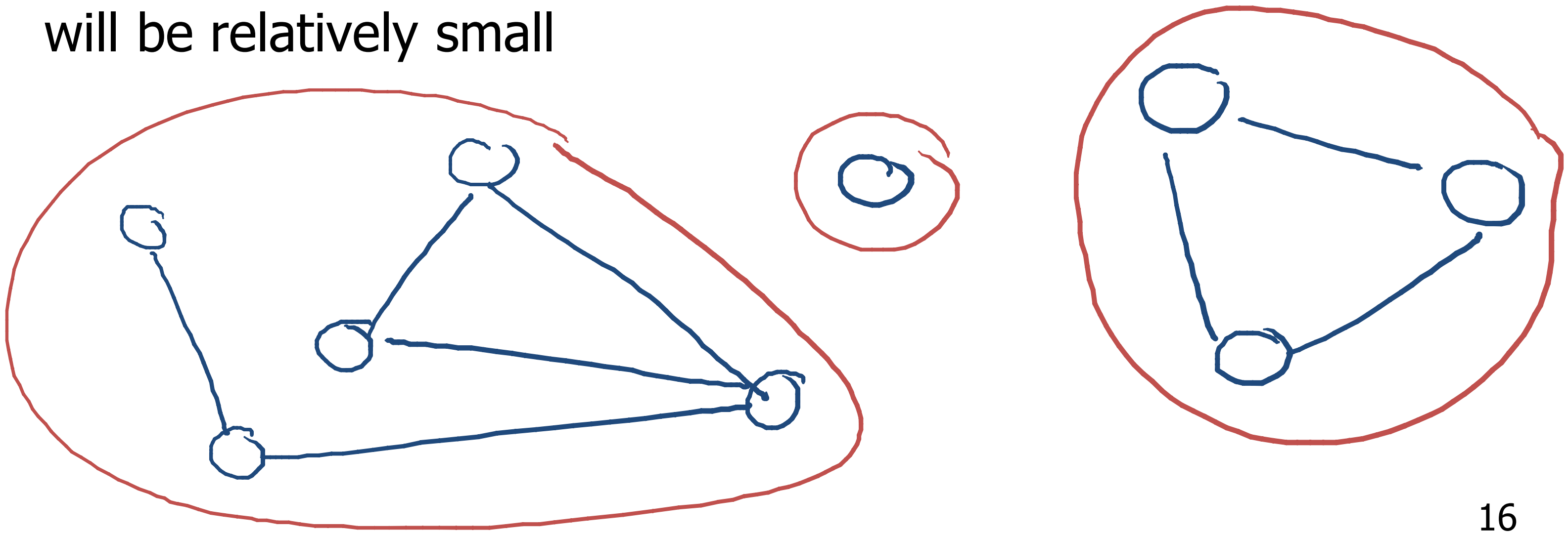
- Stopping criterion
  - It will be useful to support two modes of operation
    - stop when **a given target node** is settled
    - stop when **all reachable nodes** are settled
  - You can easily support both of these by always passing two arguments, sourceNode and targetNode, and for the second mode call with a value -1 for targetNode

Dijkstra — Implementation advice 3/3

- Standard Dijkstra requires a decrease-key operation
  - The tentative distance of a node in the priority queue (PQ) can decrease several times over the course of the execution
  - Requires an operation to decrease the key of a given PQ item
  - But PQs like the std::priority_queue, don't support this
- There is a simple trick to avoid this operation
  - Instead of a decrease-key, insert the node (again) with the smaller tentative distance
  - Whenever a node with key larger than the already known tentative distance is removed from the PQ, **ignore it**
  - Works fine as long as there are relatively few decrease-key operations, which is the case for road networks **why?**

Connected Components 1/2

- Definition
  - On an undirected graph, a connected component (CC) is a maximal subset C of nodes such that for all pairs x, y ϵ C there is a path between x and y
  - Our two OSM graphs are likely to have more than one connected component
  - But one will contain most of the nodes, and the other CCs will be relatively small



Connected Components 2/2

- Easy to compute using Dijkstra
  - Add a member variable Array<int> visitedNodes to your class DijkstrasAlgorithm, with one entry per node, and all entries initialized to 0
  - Proceed in rounds 1, 2, ... and in round i do:
    - If no more nodes are marked 0 we are done
    - Pick any node still marked 0 and run Dijkstra from that node until all nodes are settled
    - Mark all nodes visited on the way with i
  - Now it's easy to identify the connected components, and in particular the largest one

visitedNodes

1	1	2	1	3	2	1	1	1	2	2	3	1	1	3	3

0 1 2 3 4 5 6 ... 15

Jenkins

- Jenkins is a continuous build systems
  - Checks out your code from our repository
  - Does compile, test, and checkstyle
  - Makes sure that you committed all the necessary files and that everything works fine
  - Triggered by every SVN change or manually
  - If an error occurs, an email will be sent to you
  - You find the link to Jenkins on your Daphne page
  - From now on check that whatever you commit passes through Jenkins without errors, and if not correct it

References

- Dijkstra's Algorithm
  – http://en.wikipedia.org/wiki/Dijkstra's_algorithm
- Connected Components
  – http://en.wikipedia.org/wiki/Connected_component_(graph_theory)
- Jenkins
  – https://daphne.informatik.uni-freiburg.de/jenkins