Efficient Route Planning
SS 2012

Lecture 4, Wednesday May 16th, 2012
(Arc flags, Visualization)

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Overview of this lecture

- Organizational
  - Feedback and results from Exercise Sheet 3 (A-Star)
- Arc Flags algorithm
  - A method for very strong **goal direction**
  - How to compute the actual shortest path
    (that is, the arcs along the path and not just the total cost)
- Exercise Sheet 4
  - Implement a part of Arc Flags (single region) and
    - … run some queries as usual
    - … **visualize** the search space for one query

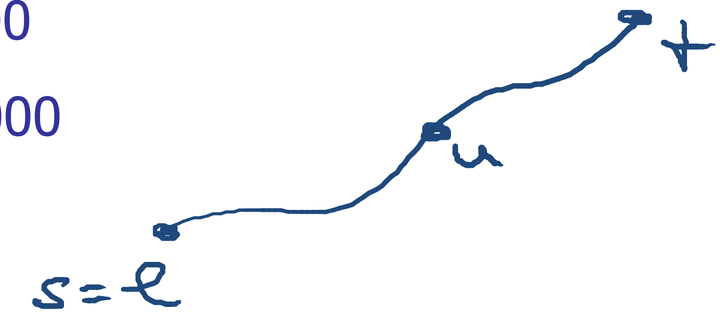
Your Feedback on Ex. Sheet 3 (A-Star)

- Summary / excerpts — last checked May 16, 16:14
  - 6 – 9 hours was typical, a few needed less, some more
  - Some used quite some extra time for refactoring old code
  - Implementation advice in the lecture was useful again
  - Feedback from the tutors was much appreciated again
  - Rounding in A-Star can impact admissability / monotonicity
    - rounding arc costs **up** always works

      $h(u) \leq c(u,v) + h(v)$
  - Question about landmarks: one standard Dijkstra per landmark, or one set Dijkstra for all landmarks together?
    - It's one Dijkstra per landmark, no set Dijkstra here
    - The set Dijkstra was just for selecting a set of landmarks that are as "far apart" from each other as possible

Experimental results from ES 3 (A-Star)

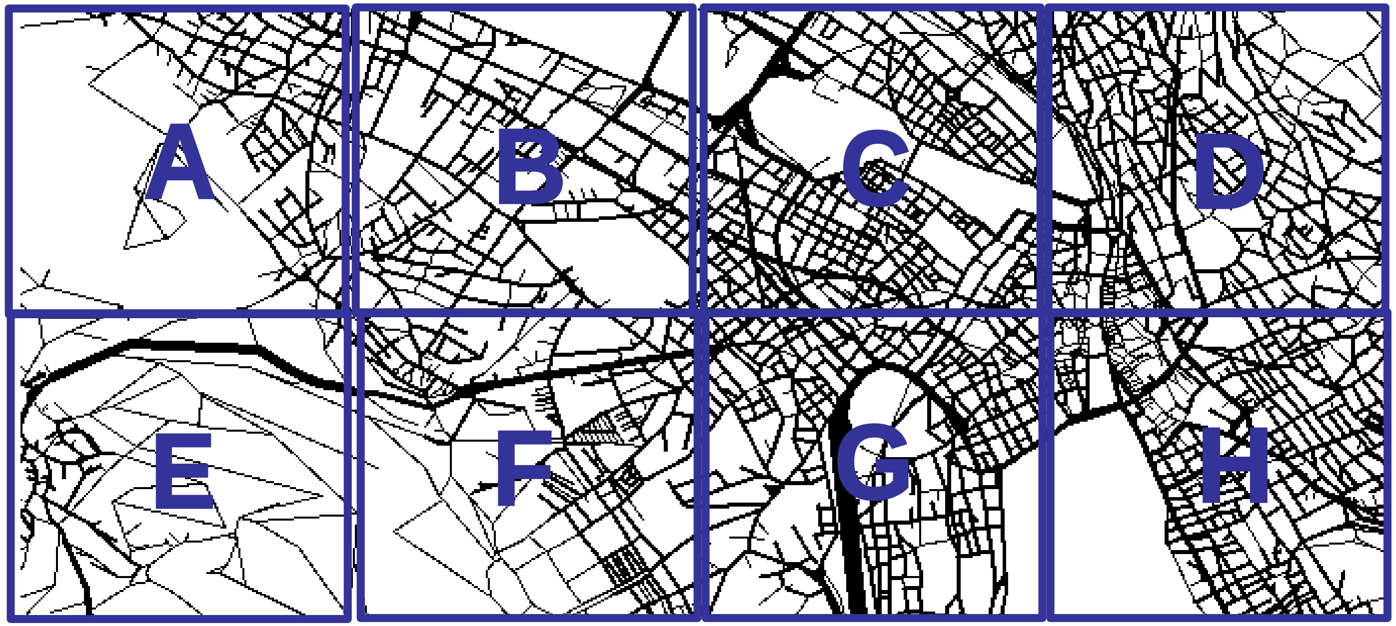
- See the table on the Wiki
  - #settled nodes (= size of search space) much decreased:
    - Dijkstra: 100,000 / 1,200,00 (Saarland / BaWü)
    - A*-Straight: 50,000 / 500,000
    - A*-Landmarks: 5,000 / 50,000
  - query times accordingly
    - Dijkstra: 20ms / 500ms
    - A*-Straight: 20ms / 250ms
    - A*-Landmarks: 1ms / 15ms
  - Bottom line: A*-Straight helps a little, A*-LMs helps **a lot**



$h_\ell(u) = |\,dist(s,t) - dist(s,u)\,|$
$= dist(u,t)$ PERFECT

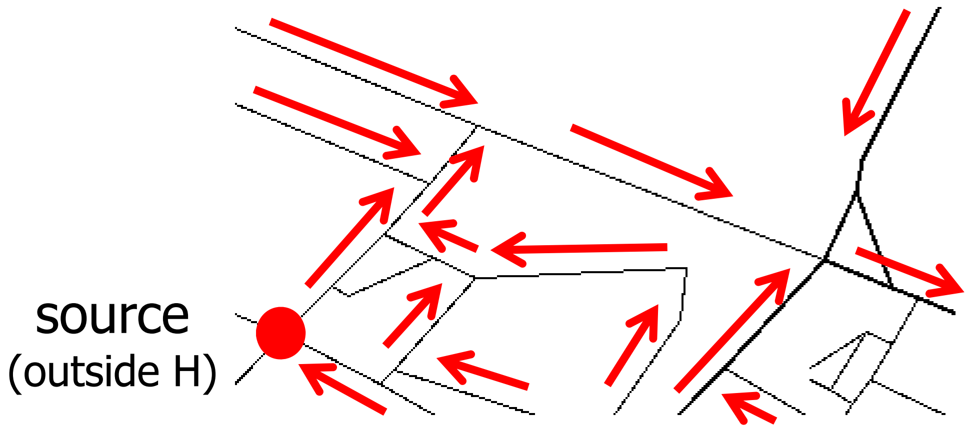
Arc flags — Basic idea 1/2

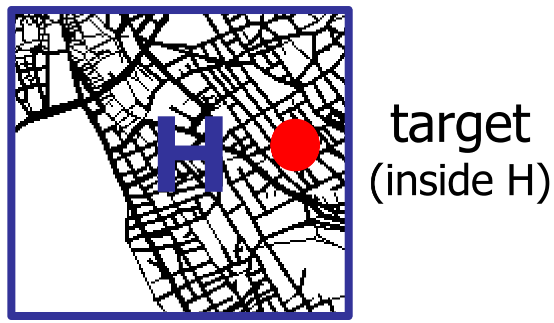
- Precomputation
  - Divide the map into "compact" regions of about equal size
  - For each arc, compute "direction signs" for each region
  - We call these direction signs **arc flags**



Arc flags — Basic idea 2/2

- At query time
  - Determine the region containing the target node
  - In Dijkstra's algorithm, **outside** of that region, consider only arcs with direction signs towards that region

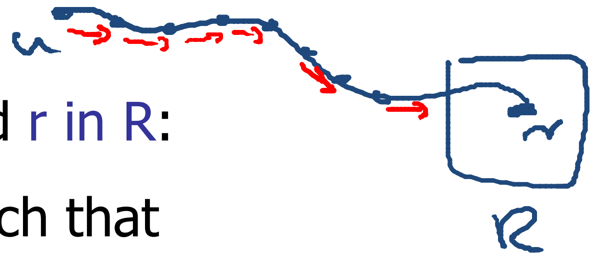




Arc Flags – Formal Definition 1/2

- Properties of the arc flags
  - Each arc has **one arc flag per region**, which is 0 or 1
  - The arc flags for a fixed region R (one per arc) must have the following properties:
    - for nodes u and r, with u arbitrary and r in R:

      there is a shortest path from u to r such that the flags of **all** arcs on that path are set to 1
    - Note: there may be several shortest path from u to r; it is enough that **one of them** has this property
  - Several ways to compute such arc flags ... later slides



Arc Flags – Formal Definition 2/2

- Query algorithm
  - Given a query from a node s to a node t, both arbitrary
  - Determine the region R containing t
    - this requires that each node lies in **some** region
  - Execute an ordinary Dijkstra on the subgraph formed by those arcs with flags for R set to 1
    - this could be implemented by making a **copy** of the graph, where we only consider those arcs
    - but it is equivalent, and more efficient, to simply **ignore** the arcs with flags for R set to 0
    - see implementation advice on later slide

Arc Flags — Correctness

- Consider a query from s to t, both arbitrary
  - Let R be the region containing t
  - Given the properties of the arc flags:
    - there exists a shortest path from s to t such that the flags for R on all arcs on that path are 1
  - Hence that path also exists in the subgraph consisting only of those arcs with flags for R set to 1
  - Since we only remove arcs and don't add any, there can't be a better path in the subgraph
  - Hence Dijkstra will find that path, or one with equal cost

Arc flags — Precomputation 1/7

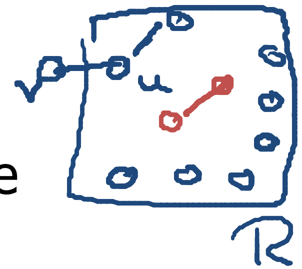
- **Naive way:** For each region R do the following
  - Do the following for **each** node r in R:
    - Run Dijkstra starting from r in the reverse graph, until all nodes (reachable from r) are settled
    - This gives us the shortest path from each node u in the graph to r ... how to obtain **paths** → later slide
    - Set flag for each arc that is on one of these paths
  - This obviously fulfills the arc flags property, recall:
    - for each u and r, with u arbitrary and r in R
    - there must be a SP from u to r with all flags set
  - The above algorithm computes one such SP for each u and r

Arc flags — Precomputation 2/7

- Cost of this naive precomputation
  - One Dijkstra for each node in each region
  - This is one Dijkstra for each node in the graph
  - The cost of each Dijkstra is $\sim m \cdot \log n$
    - where $m = \#arcs$ and $n = \#nodes$
  - This is cost $\sim n \cdot m \cdot \log n$ overall
  - Even when $m = \Theta(n)$ that is **quadratic** in n
  - That would be **infeasible** already for BaWü

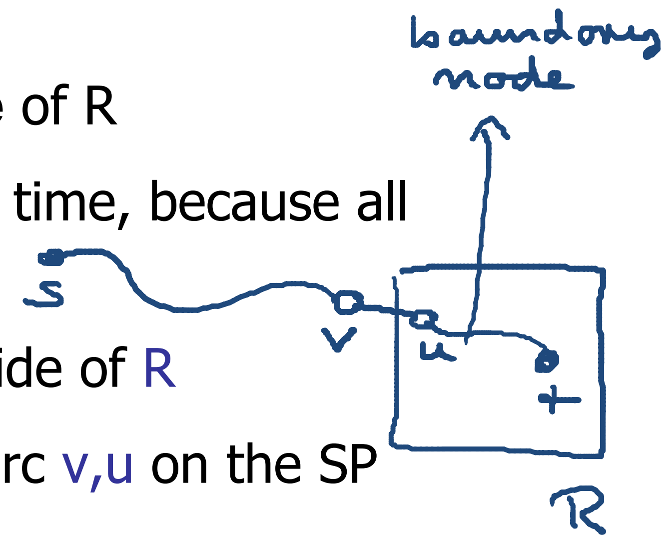
Arc flags — Precomputation 3/7

- **Better way:** For each region R do the following
  - Compute the set of boundary nodes of R:
    - a boundary node is a node u in R with at least one arc u,v such that v not in R
  - As before, but now only for each boundary node r:
    - Run Dijkstra starting from r in the reverse graph, until all nodes (reachable from r) are settled
    - Set all flags on all shortest paths thus computed
  - Additionally, set flags of all arcs u,v **inside** of R
    - an arc u,v is inside of R if **both** u and v are in R



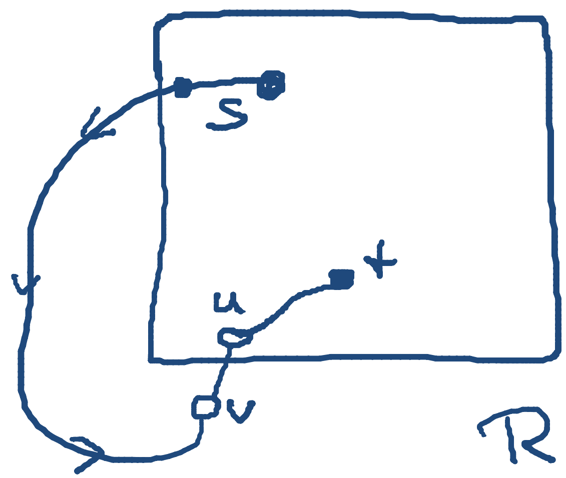
Arc flags — Precomputation 4/7

- Correctness of this "better way":
  - Consider a query from s to t, with s arbitrary and t in R
  - Consider any SP from s to t
  - **Case 1: all** arcs on that SP are **inside** of R
    - Then this SP will be found at query time, because all arcs inside of R are set
  - **Case 2:** not all arcs on that SP are inside of R
    - Since t is in R, there must be one arc v,u on the SP with v not in R and u in R
    - The subpath from s to u is a SP from s to u
    - The precomputation for u will find this path or a path of equal cost and set the flags of all arcs on it

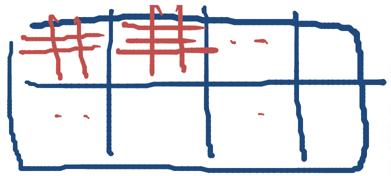


Arc flags — Precomputation 5/7

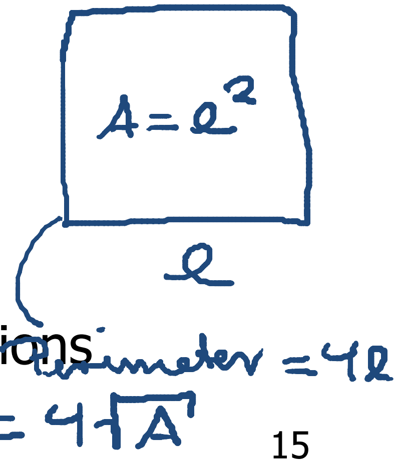
- Finer points of this argument
  - Note that even if s and t both lie in R, both cases can happen:



Arc flags — Precomputation 6/7



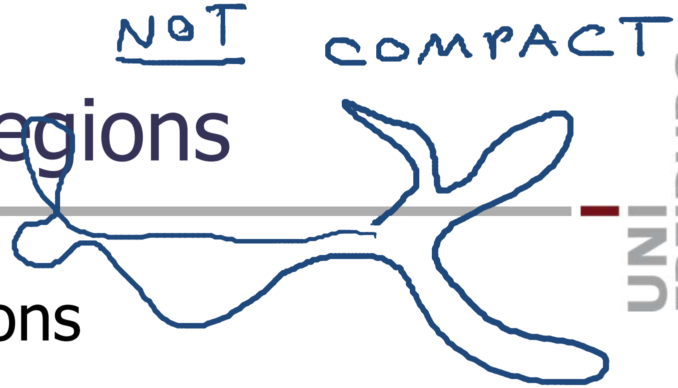
- Precomputation costs of the "better way":
  - We now have one Dijkstra **per boundary node**
  - So the total cost is $\sim b \cdot n \cdot \log m$
    where b = #boundary nodes, n = #nodes, m = #arcs
  - The size of b depends on the division into regions
  - Here is an estimate, if we divide into k **square** regions
    and assuming that the nodes are equally distributed
    - each region contains $\sim n/k$ nodes
    - of those, $\sim 4 \cdot (n/k)^{1/2}$ lie on the boundary
    - hence $b \sim 4 \cdot (n \cdot k)^{1/2}$
  - Hence total cost $\Omega(n^{3/2} \cdot \log m)$ even for $\Theta(1)$ regions



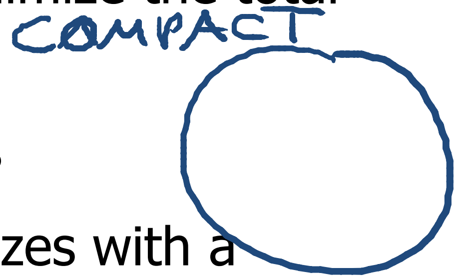
Arc flags — Precomputation 7/7

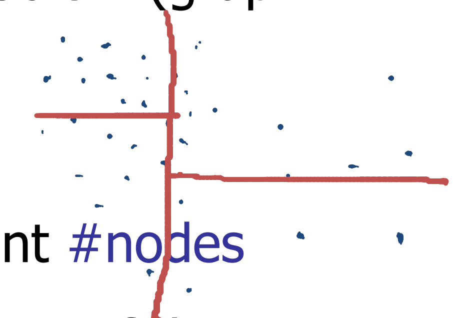
- Space consumption for storing the arc flags
  - Assume we have k regions, then we need k bits per arc
    - That is $k/8 \cdot m$ Bytes, where $m = \#arcs$
  - Let's compare that to the storage needed for the graph
    - 12 bytes per node (OSM id + latitude + longitude)
    - 8 bytes per arc (head node id + cost)
    - That is $12n + 8m$ bytes, where $n = \#nodes$, $m = \#arcs$
    - For road networks we have $m \approx 2.25n$
    - That is, we need about 13 bytes / arc for the graph
    - So for $k > 100$ the arc flags start to become expensive also storage-wise

Arc flags — Division into regions



- What is a good division into regions
  - For a fixed number of regions we want to minimize the total number of boundary nodes
    - Intuitively, this calls for "compact" regions
  - Dividing a graph into k subgraphs of similar sizes with a minimal number of boundary nodes is a hard problem (graph partitioning)
  - Rectangular regions are ok, but not optimal
    - for road networks, can contain widely different #nodes
  - Something like a KD-tree gives an even distribution of the #nodes / region, but not necessarily a small #boundary nodes

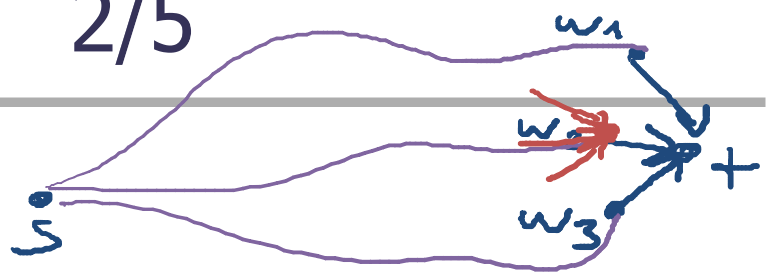




Paths, not only costs 1/5

- So far we only computed SP **costs**, not the **paths**
  - For the arc flags precomputation we need the paths
  - Any route planning system will want to output paths
  - So how do we get the actual paths?

Paths, not only costs 2/5



- There is a generic way
  - Assume we have stored the dist(s, u) for **all** nodes u that we have settled in a Dijkstra / A* computation from s to t
  - Then we can compute an SP from s to t as follows:
    - Consider the set W of all nodes w such that w was settled in the computation above and an arc w,t exists
    - Note that there will be at least one such w, namely the node from which t got its label by relaxing
    - Compute $v = \text{argmin}_{w \in W}\ \text{dist}(s, w) + \text{cost}(w,t)$
    - Then v is a predecessor on an SP from s to t
    - Now repeat with v in place of t ... until s is reached

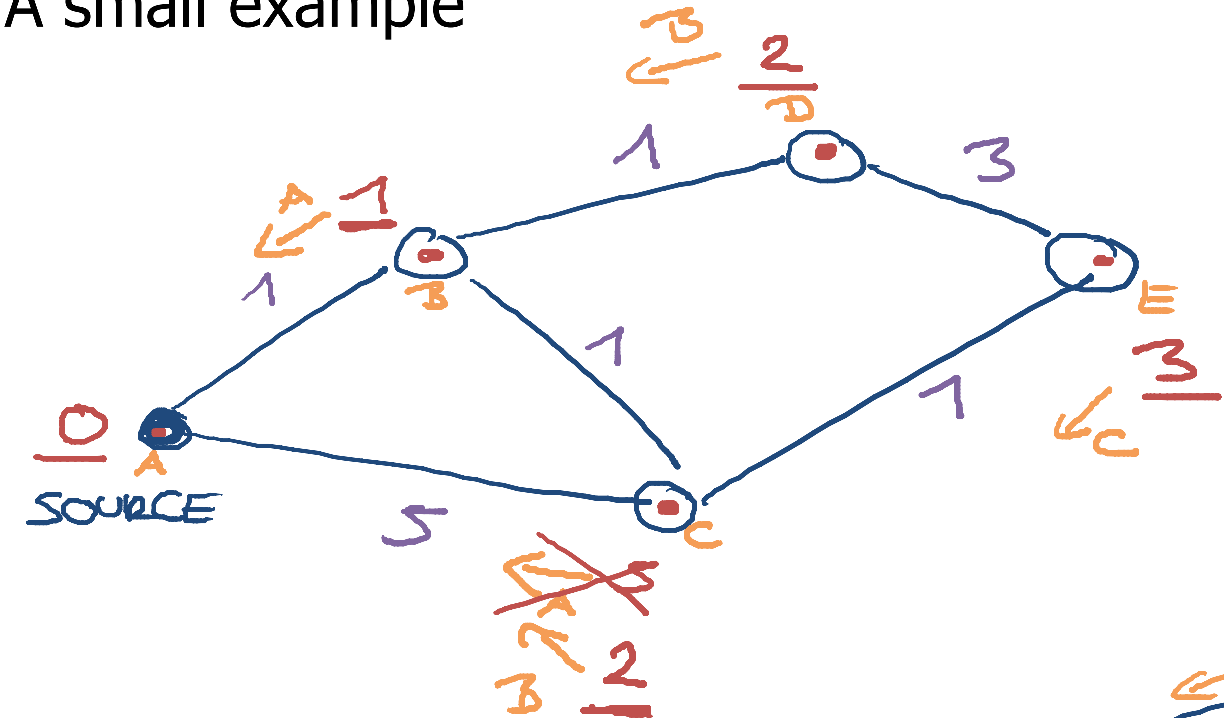
Paths, not only costs 3/5

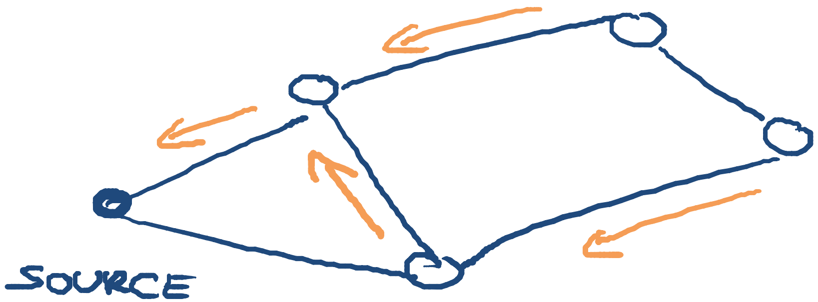
- But easier to compute this **during** Dijkstra
  - Along with the **dist** value for each node
  - Also maintain a **parent pointer** for each node
  - This is simply the id of the node from which the current dist value comes via relaxation

    (Initialize to some non node id, for example -1)
  - By the correctness proof of Dijkstra / A*, the parent pointer of each **settled** node u than points to the predecessor on a shortest path from s to u
  - That is, this pointer exactly points us to the v computed with the argmin on the previous slide

Paths, not only costs 4/5

- A small example

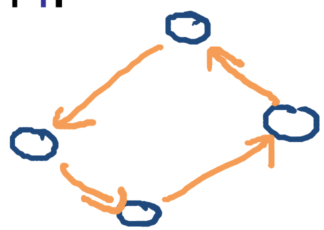




Paths, not only costs 5/5

- The parent pointers form a **tree** rooted at s
  - This can be proven by by a simple extension of our correctness proof for Dijkstra / A*

    (assuming the same order of nodes u_1, u_2, u_3, ...)
  - Namely, we can prove (by induction) that in iteration i:
    - u_i is settled
    - $dist[u_i] = dist(s, u_i)$
    - $parent[u_i]$ = the predecessor of u_i on an SP from s to u
  - This implies that there can be no cycles not containing s
  - And no cycles containing s either, because s has no parent

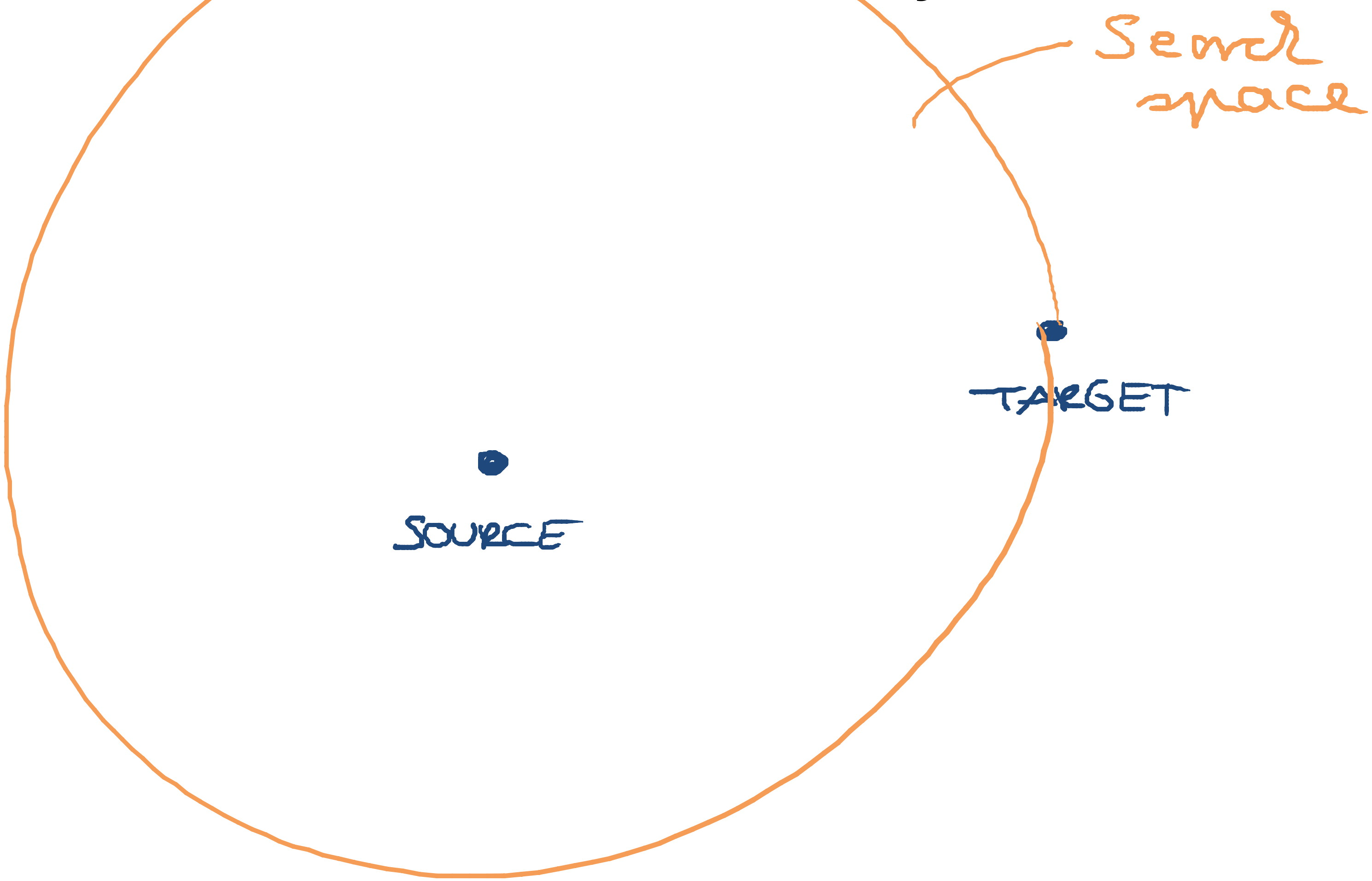


Arc Flags — Implementation Advice

- For the Ex. Sheet: a **single rectangular** region R
  - Write a new class ArcFlagsAlgorithm
  - Ok to compute the boundary nodes in the trivial way
    - iterate over all **arcs** u,v and mark u as boundary node if u in R and v not in R
  - Execute **one** Dijkstra **per** boundary node
  - Add a member variable arcFlag to your Arc class
  - Extend your class DijkstrasAlgorithm by a mode that relaxes an Arc only if the arcFlag is set ... that's trivial
  - See the code design suggestion on the Wiki
  - New stuff is commented with // NEW(lecture-4): ...

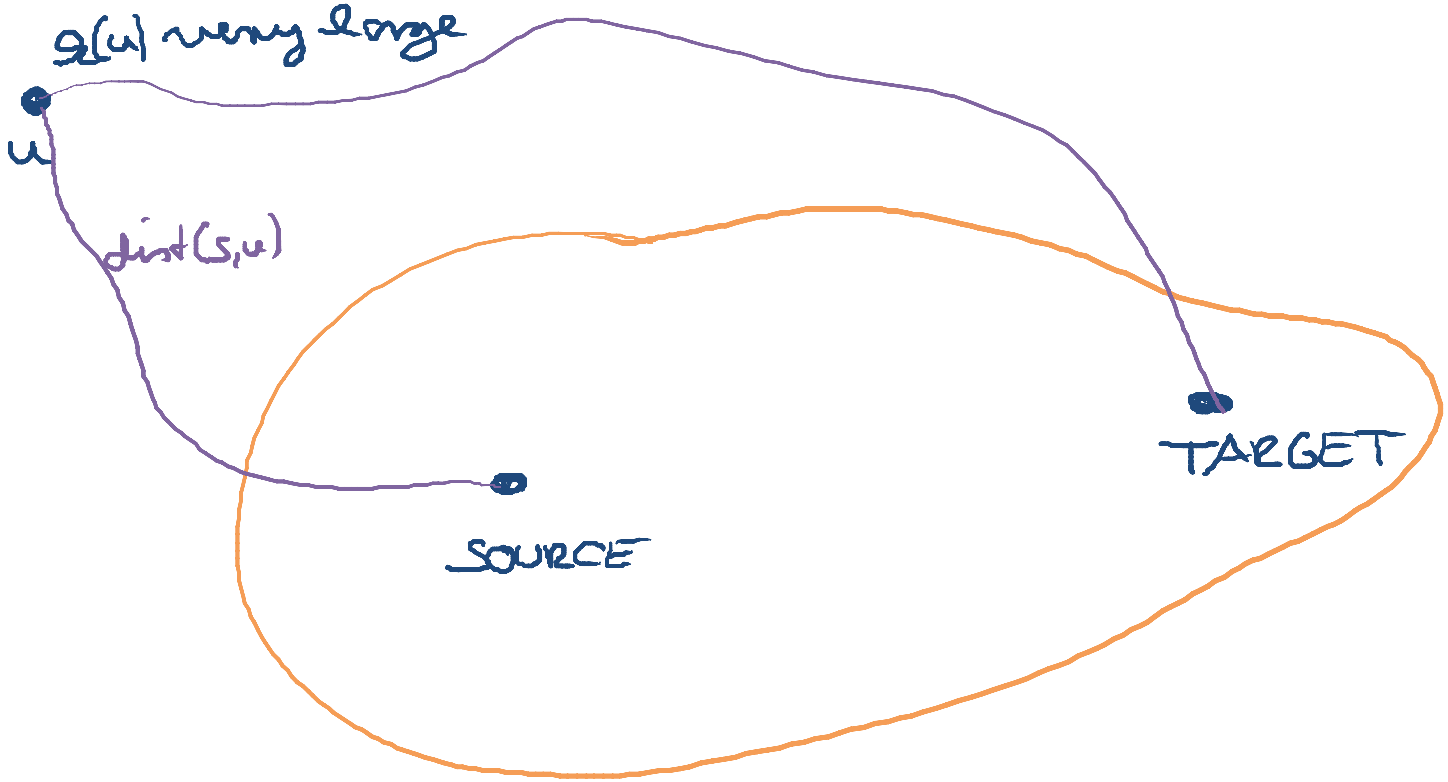
Search space comparison 1/3

- Set of settled / visited nodes for Dijkstra



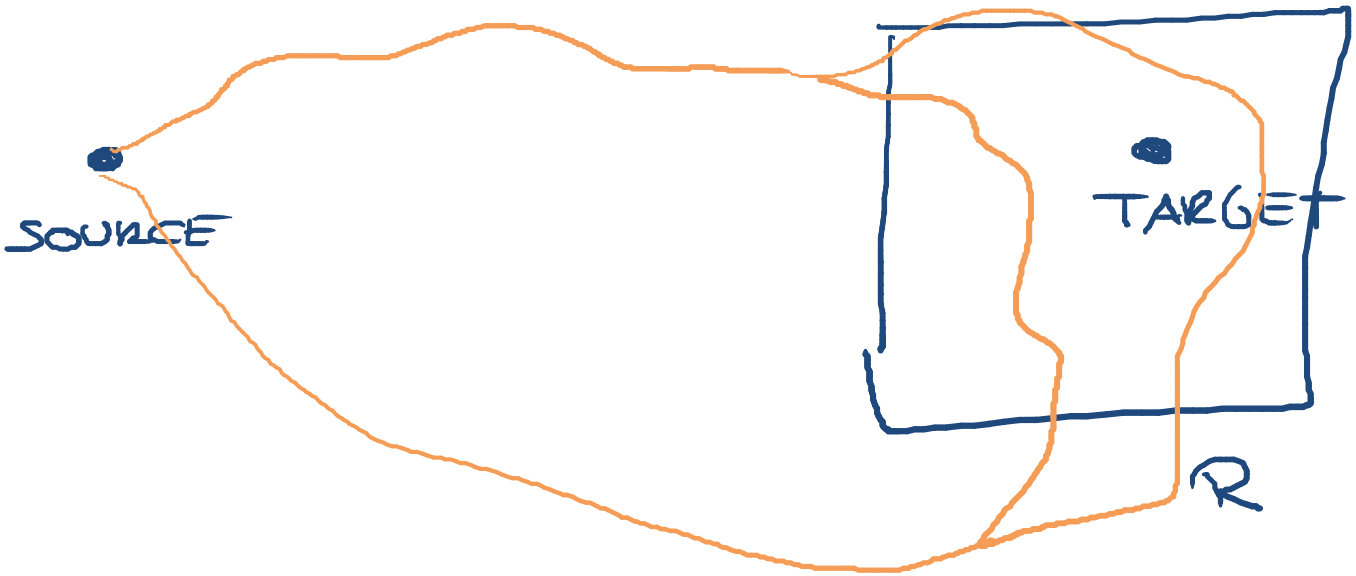
Search space comparison 2/3

- Set of settled / visited nodes for A-Star



Search space comparison 3/3

- Set of settled / visited nodes for Arc Flags



Google Fusion Tables

- Nice tool to visualize geo data on Google Maps
  - You can upload a CSV file with coordinates, e.g.

    47.95 7.75
    47.95 7.90
    48.05 7.75
    48.05 7.90

  - And then draw the points on Google Maps with one click
  - In the visualization, there is a button for a permanent link to your visualization
  - For Ex. Sheet 4: visualize the set of visited nodes for one of your queries and link to it in the result table on the Wiki
  - http://www.google.com/fusiontables

References

- First arc flag paper

  An extremely fast, exact algorithm for finding shortest paths in static networks with geographical background

  Ulrich Lauther, Münsteraner GI-Tage 2004

  https://gor.uni-paderborn.de/Members/AG06/LAUTHER.PDF

- Arc flags with various tricks + a hierarchy of regions

  Acceleration of Shortest Path and Constrained Shortest Path Computation

  E. Köhler and R. Möhring and H. Schilling, WEA 2005

  ftp://ftp.math.tu-berlin.de/pub/Preprints/combi/Report-042-2004.pdf

  http://www.springerlink.com/content/wc06qawxy5bc5bj0/