Efficient Route Planning

SS 2012

Lecture 8, Wednesday July 27th, 2012

(Transit Node Routing)

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Overview of this lecture

- Organizational
  - Feedback and results for Exercise Sheet 7 (CH, part 2)
- Transit Node Routing (TNR)
  - Last algorithm (in this course) for routing on **road** networks
  - One of the (algorithmically) fastest one to date
  - Very simple idea + very simple query algorithm
  - Various possibilities for the pre-computation … we will look at one based on Contraction Hierarchies
  - Historically TNR came two years before CH
  - Exercise Sheet 8: Implement a part of TNR

Feedback on Ex. Sheet 7 (CH, Part 2)

- Summary / excerpts   last checked June 27, 10:38
  - The way how and why Contraction Hierarchies works became much clearer in the last lecture
  - Again, not a lot of code
  - Easy to make lots of small mistakes
    - which don't show in the unit tests on small examples
    - which cause the number of shortcuts to explode in the end
  - Many could not fix all those mistakes ... frustrating

Results for Ex. Sheet 7 (CH, Part 2)

- Summary
  - Very fast precomputation: ~ 1 minute even on BaWü
  - Number of shortcuts ~ 2 million for BaWü
    - that's about the order of the number of arcs in the original graph, which is ok
  - Query times around 1 millisecond
  - All in all, clearly the best algorithm so far
  - BUT: also the hardest to implement
    - not a lot of code, but small mistakes can make everything fail ... and are hard to find (because they don't show in simple test cases)

Transit Node Routing 1/6

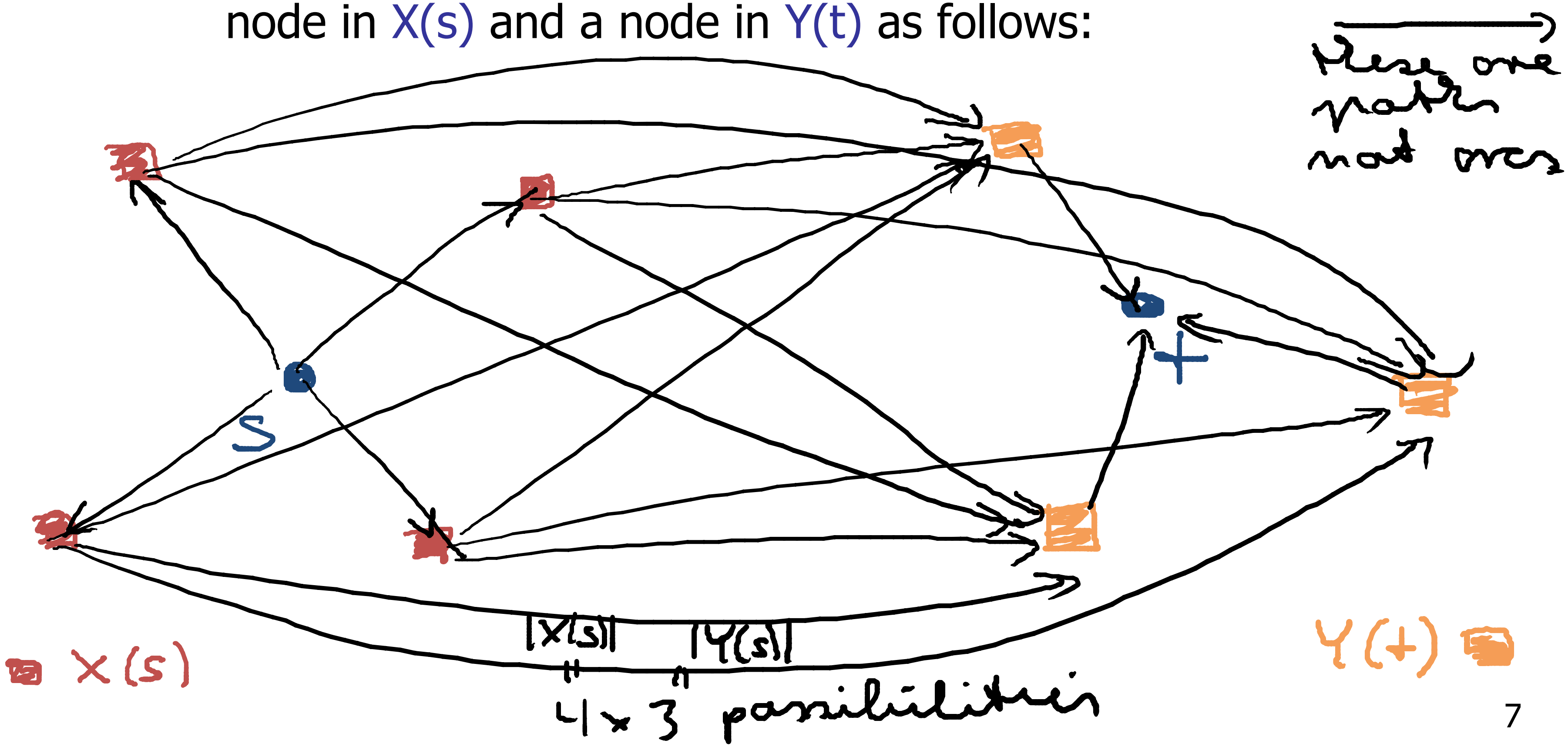
- The underlying very simple observation
  - When you go from your home to somewhere far away:
    - then the initial portion of your route will be one of a few standard routes
  - Let's look at a few examples on Google Maps
  - How can we use this to speed up shortest path queries?

Transit Node Routing 2/6

- We want to have the following
  - For each pair of nodes u and v a **"far-away" criterion** Far(u, v) that yields true or false
    - Intuitively, if Far(u, v) = true then u and v are "far away"
  - For each node u sets X(u) and Y(u) of **access nodes** such that
    - For all v with Far(u, v) = true → exists x ∊ X(u) on SP(u,v)
    - For all w with Far(w, u) = true → exists y ∊ Y(u) on SP(w, u)
    - Intuitively: when you go **from** u to somewhere "far away", you will pass through one of the X(u) ... and same for Y(u) when you go **to** u from somewhere "far away"
  - Note: for symmetric graphs, going from and going to is the same and X(u) = Y(u)

Transit Node Routing 3/6

- Processing a query from s to t — Basic idea:
  - If Far(s, t) = false (close together) ... use any algorithm
  - If Far(s, t) = true (far away) ... try all combinations of a node in X(s) and a node in Y(t) as follows:



Transit Node Routing 4/6

- Precomputation — Basic idea
  - Compute "something" such that $Far(u, v)$ can be evaluated quickly for given u and v
  - Compute and store the $X(u)$ and $Y(u)$ for each node u, as well as $dist(u, x)$ for each $x \in X(u)$ and $dist(y, u)$ for each $y \in Y(u)$
    - These are $\Sigma_u (|X(u)| + |Y(u)|)$ nodes and distances
  - Compute and store the unions $X = \bigcup_u X(u)$ and $Y = \bigcup_u Y(u)$ and the $dist(x, y)$ for all pairs x and y with $x \in X$ and $y \in Y$
    - These are $|X| \cdot |Y|$ distances
    - Our goal will be that both $|X|$ and $|Y|$ are on the order of $\sqrt{n}$ and not n, so that $|X| \cdot |Y| = O(n)$

Transit Node Routing 5/6

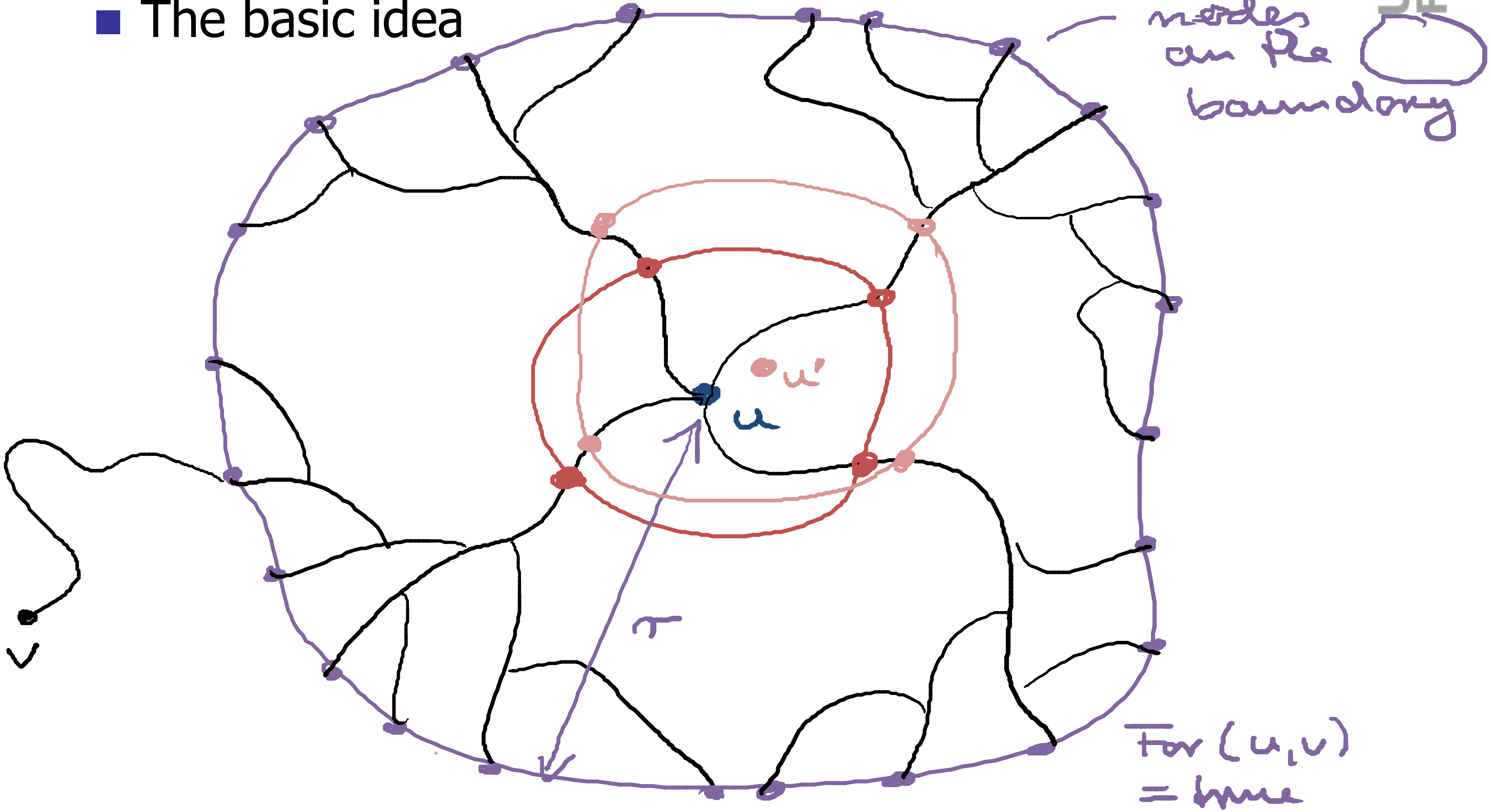
- Processing a query from s to t — Details
  - If Far(s, t) = false, compute dist(s, t) with another algorithm, for example ordinary Dijkstra; otherwise:
  - Fetch the set X(s) and the dist(s, x) for all $x \in X(s)$
  - Fetch the set Y(t) and the dist(y, t) for all $y \in Y(t)$
  - Fetch the d(x, y) for all x, y with $x \in X(s)$ and $y \in Y(t)$
  - Compute the minimum dist(s, x) + dist(x, y) + dist(y, t) over all x, y with $x \in X(s)$ and $y \in Y(t)$
    - this is the minimum over $|X(s)| \cdot |Y(t)|$ terms
    - in practice $|X(s)|$ and $|Y(t)|$ can be made as small as **5** on average ... this gives extremely fast query times

Transit Node Routing 6/6

- Efficiency
  - **Goal 1:** Far(u, v) should be very cheap to evaluate, and if Far(u, v) = false then SP(u, v) should be very cheap to compute
    - Then we can easily determine whether we have to resort to the fallback algorithm, and if so, it will be very cheap
  - **Goal 2:** X(u) and Y(u) should be ≤ a small C for (almost) all u
    - Then the X(u) and Y(u) and the distances to / from them can be stored in $\sim C \cdot n$ space, and queries can be processed in time C^2
  - **Goal 3:** $|X = \cup_u X(u)|$ and $|Y = \cup_v Y(v)|$ are $O(\sqrt{n})$
    - then the pairwise distances dist(x, y) with $x \in X$ and $Y \in Y$ can be stored in $O(n)$ space

Geometric Precomputation 1/3

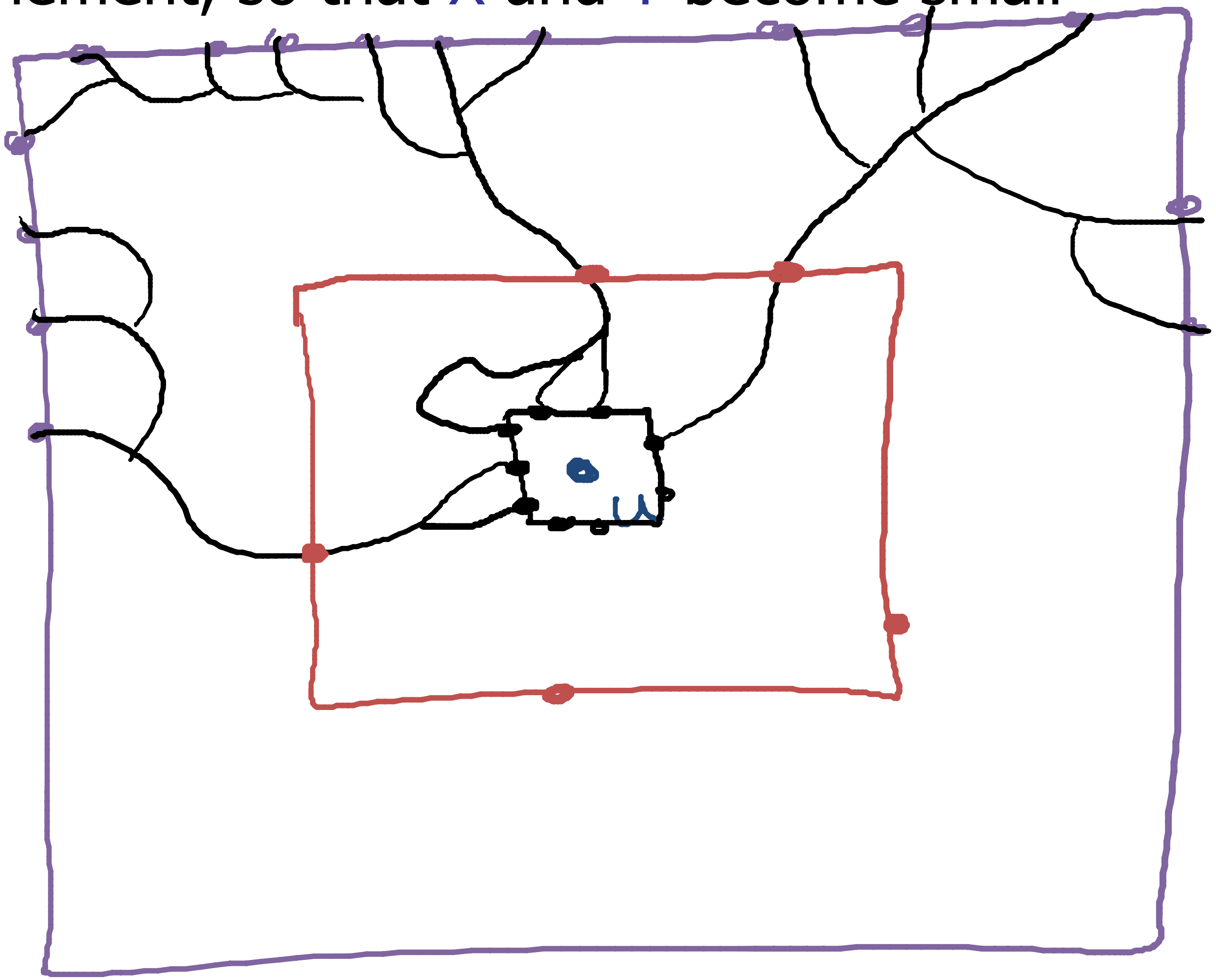
- The basic idea



For (u,v) = true
⟺ |uv| > r

Geometric Precomputation 2/3

- Refinement, so that X and Y become small



Geometric Precomputation 3/3

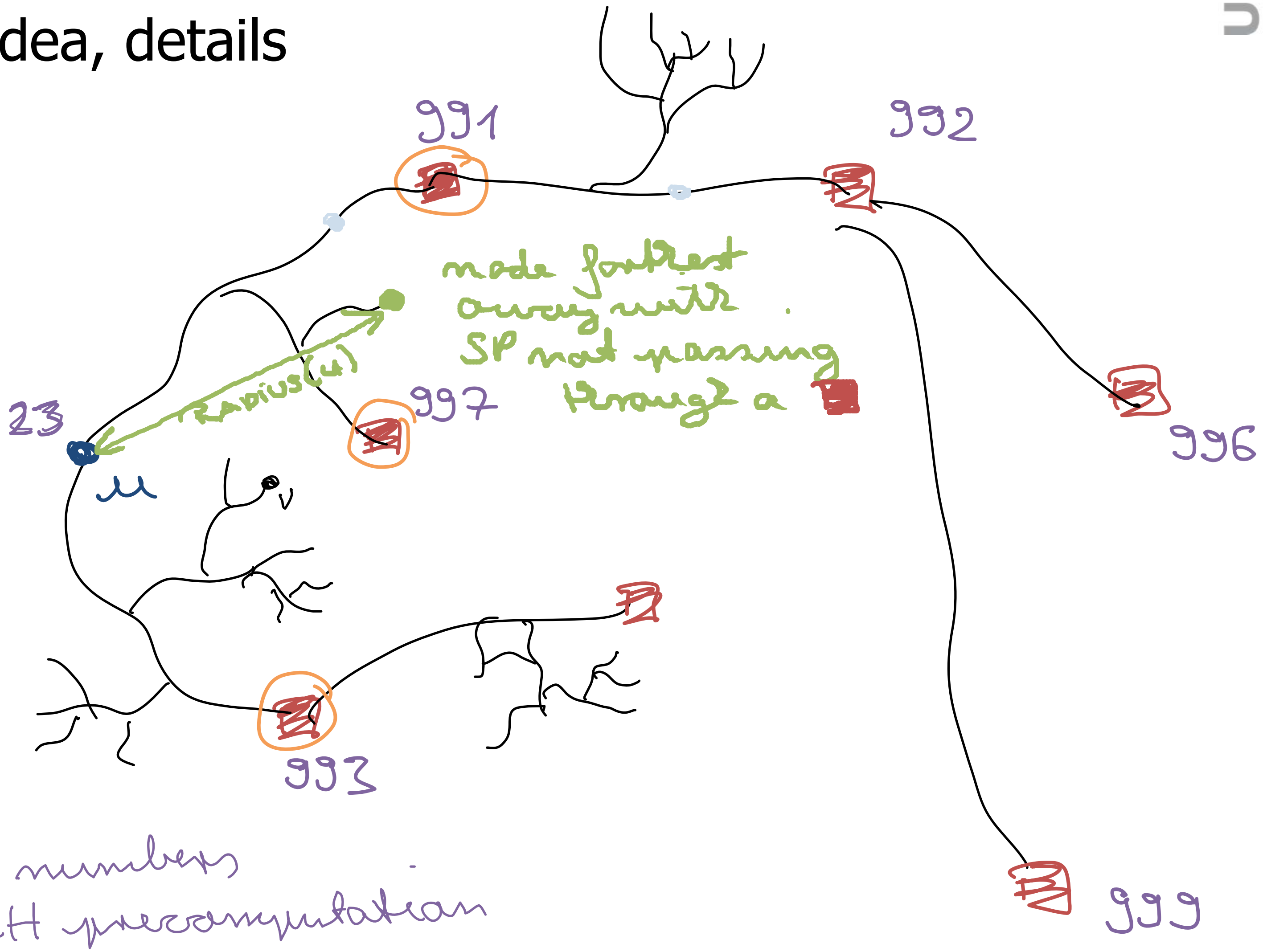
- Resource requirements
  - Small sets of access and transit nodes
  - But precomputation time comparable to that for arc flags
    (we need a Dijkstra for each boundary node of each cell)
  - There are various tricks to make this faster
  - And we can also make it hierarchical ... see later slide
  - See the references for details
  - But let's now look at a precomputation based on CH

Precomputation based on CHs 1/4

- Basic idea
  - Do the CH precomputation on the given graph
  - Let $X = Y$ be the set of nodes with ordering number above a certain threshold T (we want $|X| = |Y| \sim \sqrt{n}$)
  - For each node u in the graph do a forward search in the upward graph, and for each settled node v compute the first node $x \in X$ on $SP(u, v)$ if any; let $X(u)$ be the union of all these x
  - Similarly, compute $Y(v)$ for each node v in the graph via a backward search in the downward graph

Precomputation based on CHs 2/4

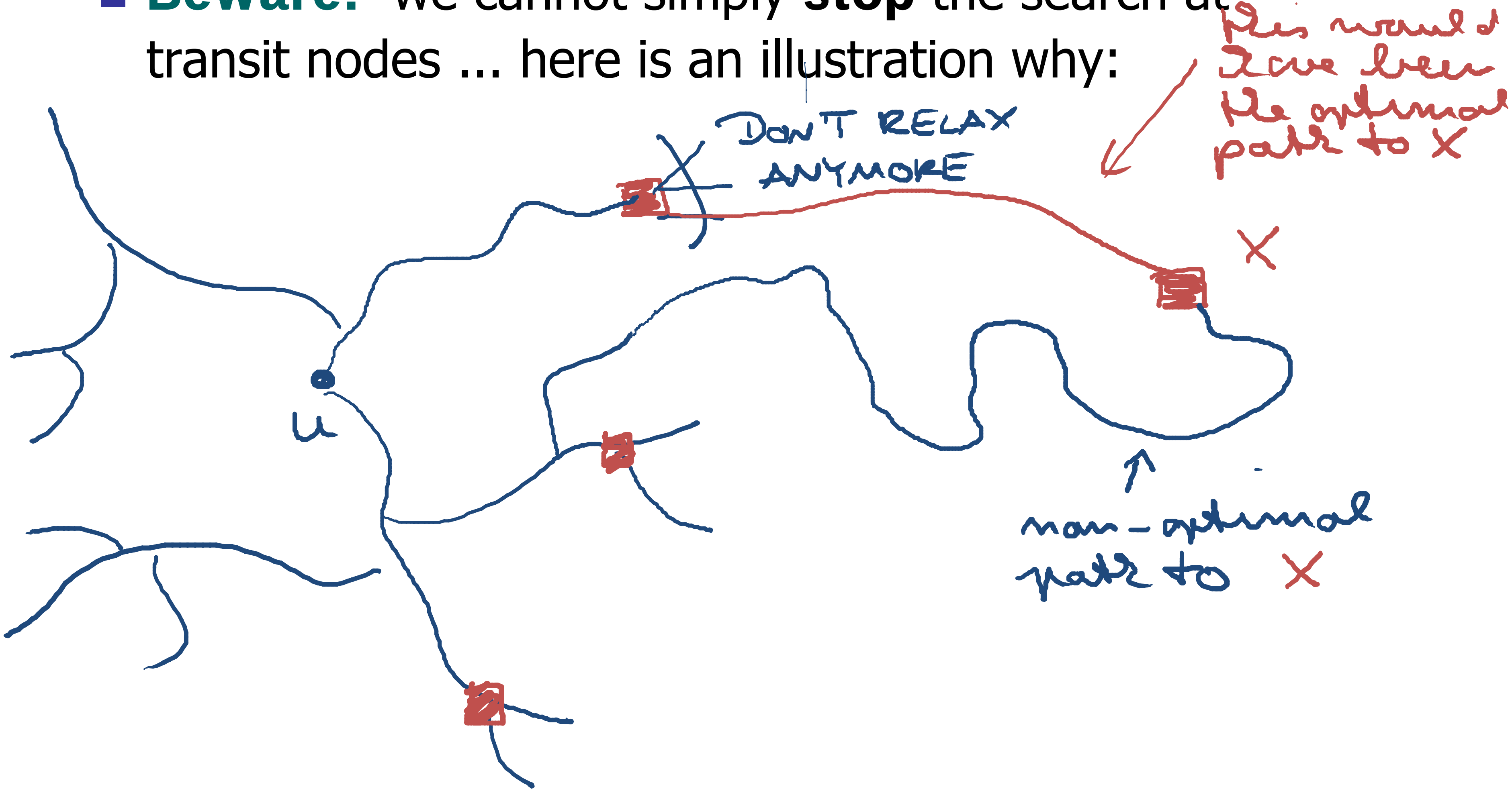
- Basic idea, details



ordering numbers
of the CH precomputation

Precomputation based on CHs 3/4

- **Beware:** we cannot simply **stop** the search at transit nodes ... here is an illustration why:



Precomputation based on CHs 4/4

- "Far-away" criterion
  - Along with the computation of X(u) ... see picture on slide 15
    - Compute the maximal geometric distance Radius(u) of a node v where SP(u, v) does **not** contain a node from X(u)
  - Define Far(u, v) = true if and only if the geometric distance from u to v is > Radius(u)
  - We can also do the same for Y(v) and thus possibly further improve our "far-away" criterion
  - For more refined "far-away" criteria, see papers in references
    - **Note:** the "far-away" criterion is called locality criterion there with exactly the opposite meaning ... quite confusing

Implementation advice 1/2

- Precomputation
  - Just do the CH precomputation and pick as transit nodes the T nodes contracted last ... it's really that simple
- Access nodes
  - For symmetric graph, X(u) = Y(u), that is we only need to compute **one** set of access nodes per set
  - For each u, you need to find the transit nodes on **all** shortest paths starting at u (in the upwards graph)
  - For each settled label, just **backtrack** the parent pointers
  - Beware: no need to backtrack further from a node which you have already seen before in the backtracking ... complexity should be #arcs in the SP tree

Implementation advice 2/2

- Simplification for Ex. Sheet 8
  - For a fully functional TNR you would need to precompute and store **all** $X(u)$ and **all** $dist(u, x), x \in X(u)$ to them
  - Similarly you would need to precompute and store **all** $dist(t_1, t_2)$ for **all** pairs of transit nodes t_1 and t_2
  - For BaWü you would probably run into memory problems
  - Instead do the following at query time, for given s and t
    - compute $X(s)$ and all $dist(s, x), x \in X(s)$
    - compute $X(t)$ and all $dist(x, t), x \in X(t)$
    - compute all $dist(x_1, x_2)$ where $x_1 \in X(s)$ and $x_2 \in X(t)$
  - But ignore the time for these three items when you benchmark the query time

Hierarchical TNR (sketch only) 1/2

- TNR can be made hierarchical, too
  - Here is an explanation for two levels of transit nodes
  - For each node, precompute and store the distances to the "closest" level-1 transit nodes (that is, the first level-1 transit nodes on paths to anywhere else)
  - For each level-1 transit node, precompute and store the distances to the "closest" level-2 transit nodes
  - Precompute and store the distances between all pairs of level-2 transit nodes
  - For a query from s to t, now try all combination of $(s, x_1, x_2, y_2, y_1, t)$, where x_1 and y_1 are the level-1 access nodes of s and t, respectively, and x_2 and y_2 are the level-2 access nodes of the respective x_1 and y_1

Hierarchical TNR (sketch only) 2/2

- Why does this make sense?
  - We need the pairwise distances only for the level-2 transit nodes
  - Therefore we can have more level-1 transit nodes and hence a better locality criterion = local searches needed only when s and t are very close together
  - But we have to try out more combinations at query time
  - Can be generalized to an arbitrary number of levels
  - Experiments suggest 5 levels for the road network of a whole continent (Western Europe or the US)
  - See the references for details

References

- Transit Node Routing, original paper

  Ultrafast Shortest-Path Queries Via Transit Nodes

  Bast, Funke, Matijevic, DIMACS Shortest Path Challenge

  http://www.mpi-inf.mpg.de/~bast/papers/transit-dimacs.pdf

- Transit Node Routing, based on HH and CH

  PhD thesis from Dominik Schultes (HH), Chapter 6

  http://algo2.iti.kit.edu/schultes/hwy/schultes_diss.pdf

  Master thesis from Robert Geisberger (CH), Section 4.2

  http://algo2.iti.kit.edu/documents/routeplanning/geisberger_dipl.pdf

- Transit Node Routing, article in Science Magazine

  Fast Routing in Road Networks with Transit Nodes

  http://www.sciencemag.org/content/316/5824/566.short