Efficient Route Planning

SS 2012

Lecture 6, Wednesday June 6th, 2012
(Contraction Hierarchies, Part 1 of 2)

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Overview of this lecture

- Organizational
  - Feedback and results from Exercise Sheet 5 (Web app)
- Contraction Hierarchies (CHs)
  - Yet another (clever) algorithm for fast route planning
  - Basic idea: far away from source / target only use "important" roads (think of highways)
  - This lecture: outline + the central "contraction" procedure
  - Next lecture: missing details, so that you know how to build a route planner based on CH
  - Exercise Sheet 6: implement the central **contraction** method

    (that will be the basic building block of the CH pre-processing)

Your Feedback on Ex. Sheet 5 (Web app)

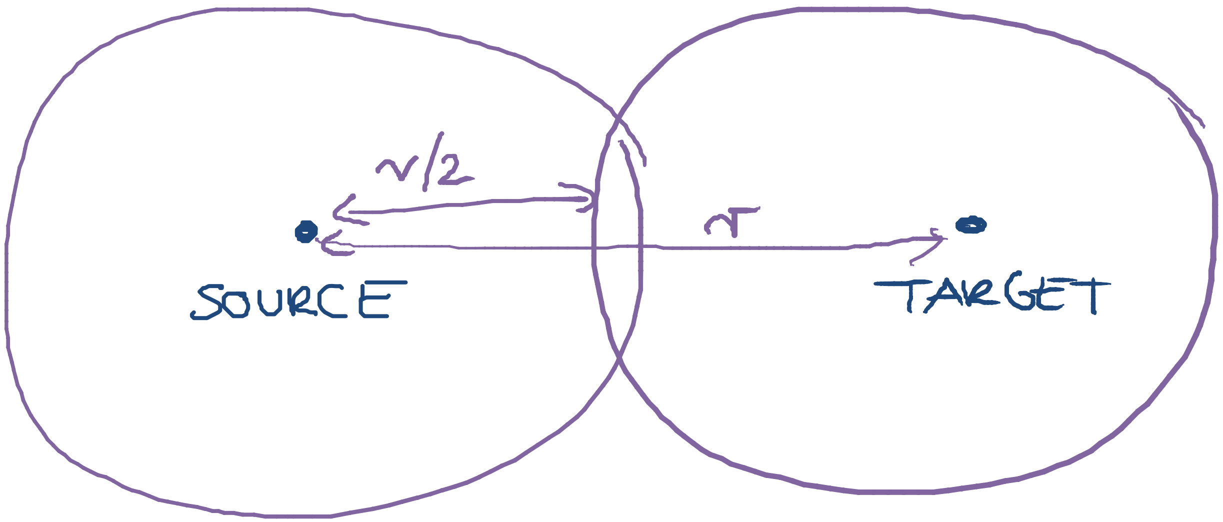
- Summary / excerpts last checked June 6, 15:51
  - Fun exercise / interesting to see how web apps work
  - Nice to see our algorithms in action / that it really works
  - Server side was relatively straightforward
    - though some used the opportunity for further imprvments
  - Client side was not hard, but quite a lot of new stuff
    - code provided was (of course) very useful
    - though one said it made thing too easy
  - Typical time investment 4-6 hours / student

Your web applications

- Let's have a look at a few demos
  - One with comparison to Google API
    - Observation: both routes reasonable, but often different
    - Reason: Google seems to penalize certain turns
  - One on Baden-Wü**rttem**berg (not Baden-Wü**rrten**berg)
    - Observation: Query time independent of dist(s, t)
    - Reason: Heuristic function computed for **all** nodes

Bidirectional Dijkstra 1/4

- Basic idea
  - "Simultaneously" search from both source and target
  - Stop when the search spaces "meet"
  - This reduces the search space only by a factor of ~ 2
  - However: bi-directional search is an important ingredient in many of the more sophisticated algorithms ... like CH



$A_r = r^2$

$2A_{r/2} = 2 \cdot \left(\frac{r}{2}\right)^2 = \frac{r^2}{2}$

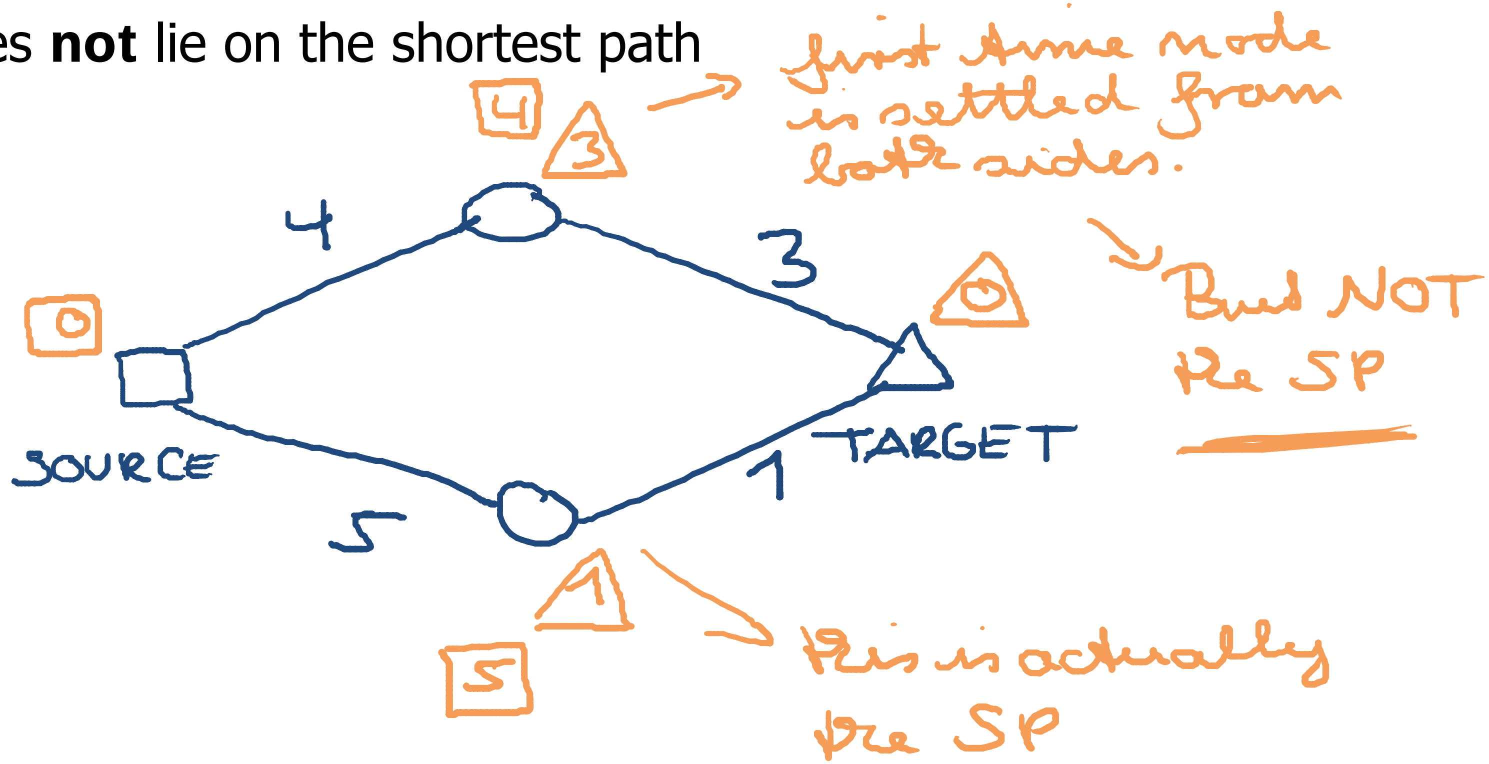
Bidirectional Dijkstra 2/4

- Implementation
  - **Interleave** the two Dijkstra computations as follows
    - in each step, one iteration from the Dijkstra where the smallest key in the PQ is smaller
    - alternatively, maintain a **joint** priority queue, where each item in the PQ knows to which Dijkstra it belongs
  - **Stop** when settling a node from one Dijkstra that is already settled in the other Dijkstra
    - that node is is **not** necessarily on the SP ... next slide
  - The cost of the shortest path is then

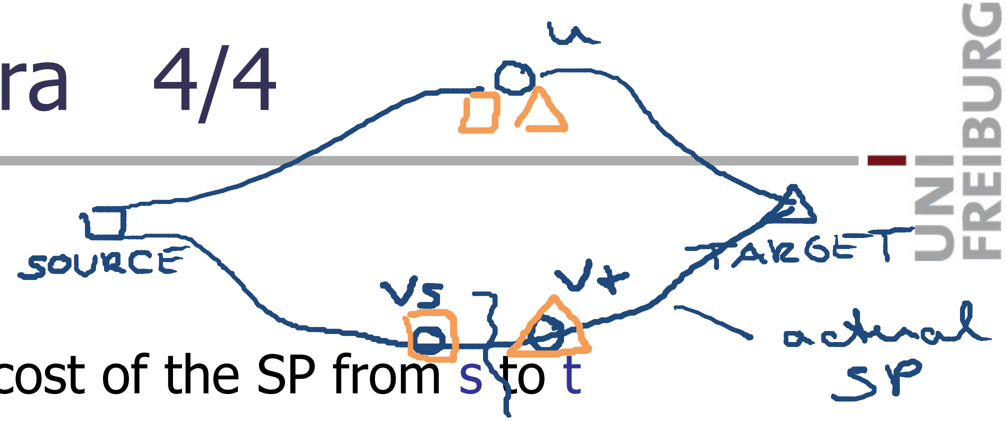
    min $\{dist_s[u] + dist_t[u]$: for all u visited in both Dijkstras$\}$

Bidirectional Dijkstra 3/4

- Counterexample
  - ... where the first node that is settled in both searches does **not** lie on the shortest path



Bidirectional Dijkstra 4/4



- Correctness proof
  - Let $D = dist(s, t)$, the cost of the SP from s to t
  - Let u be the first node settled in both Dijkstras
  - If both dist labels of u are exactly $D/2$, we are done
  - If not, one of the dist labels must be $> D/2$
  - Hence all nodes with $dist \leq D/2$ have already been settled
  - Let v_s and v_t on a shortest path from s to t such that

    $dist(s, v_s) \leq D/2$ and $dist(v_t, t) \leq D/2$
  - Then v_s has already been settled in the Dijkstra from s, and the relaxation has set $dist_s[v_t] = dist(s, v_t)$
  - Same for v_t, hence $dist_s[v_t] + dist_t[v_t] = dist(s,t)$

Hierarchical Approaches 1/4

- Basic intuition
  - "Far away" from the source and target, consider only "important" roads ... the further away, the more important
  - Let's look at the shortest path of some random queries on Google Maps, typically:

    close to source and target: mainly **white** (residential) roads

    a bit further away: mainly **yellow** (national) roads

    even further away: mainly **orange** (motorway) roads
  - But also note that this is not always true

Hierarchical Approaches 2/4

- This intuition leads to the following **heuristic**
  - Indeed consider the types / colors from the road, with an order between them, e.g. white < yellow < orange
  - Have a radius for each color > white: r_{yellow}, r_{orange}
  - Run a bi-directional Dijkstra, with the following **constraints**
    - at distance $\geq r_{yellow}$ from source and target, consider only roads of type $\geq$ yellow
    - at distance $\geq r_{orange}$ from source and target, consider only roads of type $\geq$ orange
  - **Note:** this does not necessarily find the shortest path
  - Still, heuristics of this kind were employed in navigation devices for a long time ... since no better algo's were known

Hierarchical Approaches 3/4

- Highway Hierarchies (HHs)
  - **Compute** a level for each **arc**
  - Along with a radius for each level: r_1, r_2, r_3, ...
  - Similarly as for the heuristic, run bi-directional Dijkstra
    - constraint now: at distance $\geq r_i$ from the source and target, consider only arcs of level $\geq i$
  - This was first made precise in an ESA 2005 paper by Schultes and Sanders (KIT, Karlsruhe) ... see references
  - **Note:** the basic idea is simple, but the (implementation) details are quite intricate, in particular:
    - hard to get the implementation error-free in practice

Hierarchical Approaches 4/4

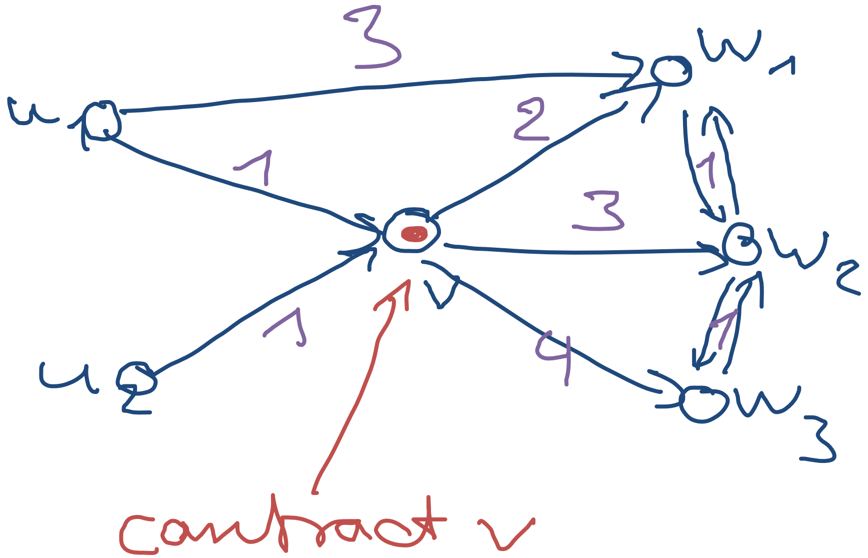
- Contraction Hierarchies (CHs)
  - **Compute** a level for each **node**
  - At query time again do a bidirectional Dijkstra
    - in the Dijkstra from the source consider only arcs u,v where level(v) > level(u) ... so called **upwards** graph
    - in the Dijkstra from the target, consider only arcs v,u with level(v) > level(u) ... so called **downwards** graph
  - Intuitively, this is like a "continuous" version of highway hierarchies ... and significantly easier to implement
  - We will look at CH in more detail now ...

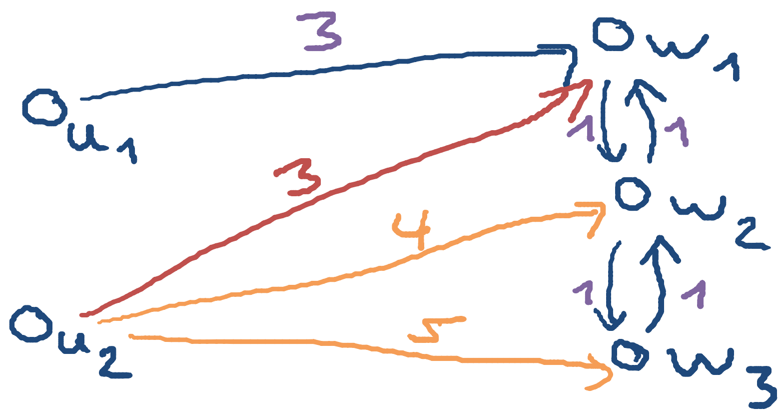
CH — Precomputation 1/4

- Contraction of a single node
  - This is the basic building block of the CH precomputation
  - Idea: take out a node, and add all necessary arcs such that **all** SP distances in the remaining graphs are preserved
  - Formally, a node v is **contracted** as follows
    - Let $\{u_1,\ldots,u_l\}$ be the incoming arcs, i.e. $(u_i, v) \in E$
    - Let $\{w_1,\ldots,w_k\}$ be the outgoing arcs, i.e. $(v, w_j) \in E$
    - For each pair $\{u_i, w_j\}$, if (u_i, v, w_j) is the **only** shortest path from u_i to w_j, add the **shortcut** arc (u_i, w_j)
    - Then **remove** v and its adjacent arcs from the graph

CH — Precomputation 2/4

- Example for contraction of **a single node**





— shortcut **must** be added

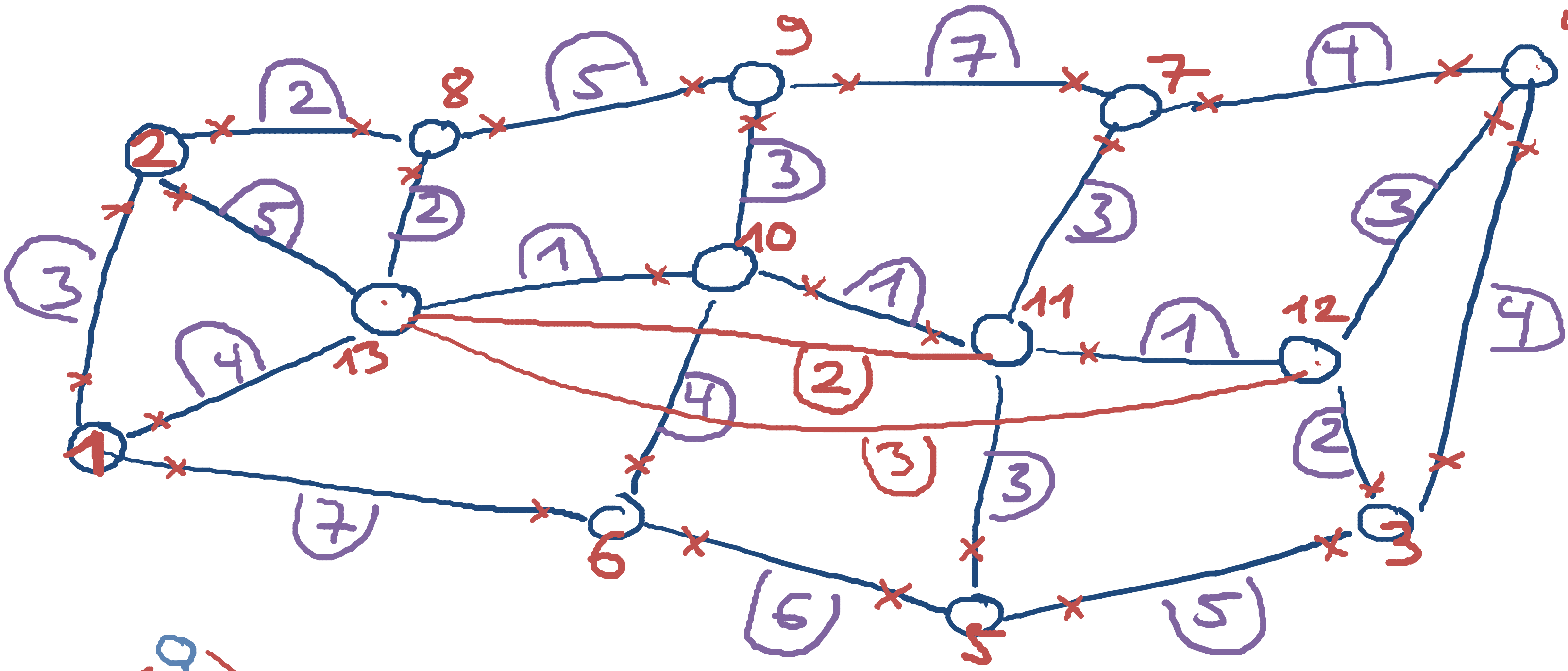
— shortcut not absolutely necessary, but OK to add it ... see later slide

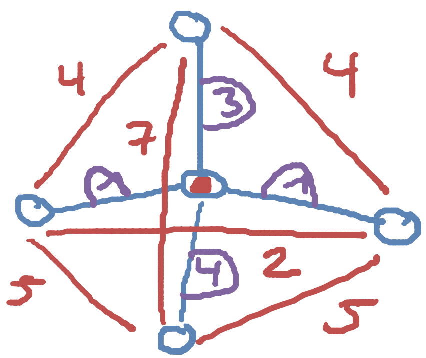
CH — Precomputation 3/4

- Contraction of all nodes in the graph
  - Let $u_1, \ldots, u_n$ be an **arbitrary** order of the nodes
  - We will see that CH is correct for **any** order, but more efficient for some orders than for others ... next lecture
  - Let $G = G_0$ be the initial graph
  - Let G_i be the graph obtained from G_{i-1} by contracting u_i that is, **without** u_i and adjacent arcs and **with** shortcuts
    - in particular therefore, G_i has $n - i$ nodes
  - In the end, let G^* = the original graph with **all nodes and arcs** and **all shortcuts** from any of the $G_1, G_2, \ldots$
  - In the implementation, we can work on **one and the same** graph data structure throughout the algorithm ... later slide

CH — Precomputation 4/4

- Example for contraction of **all nodes** in a graph





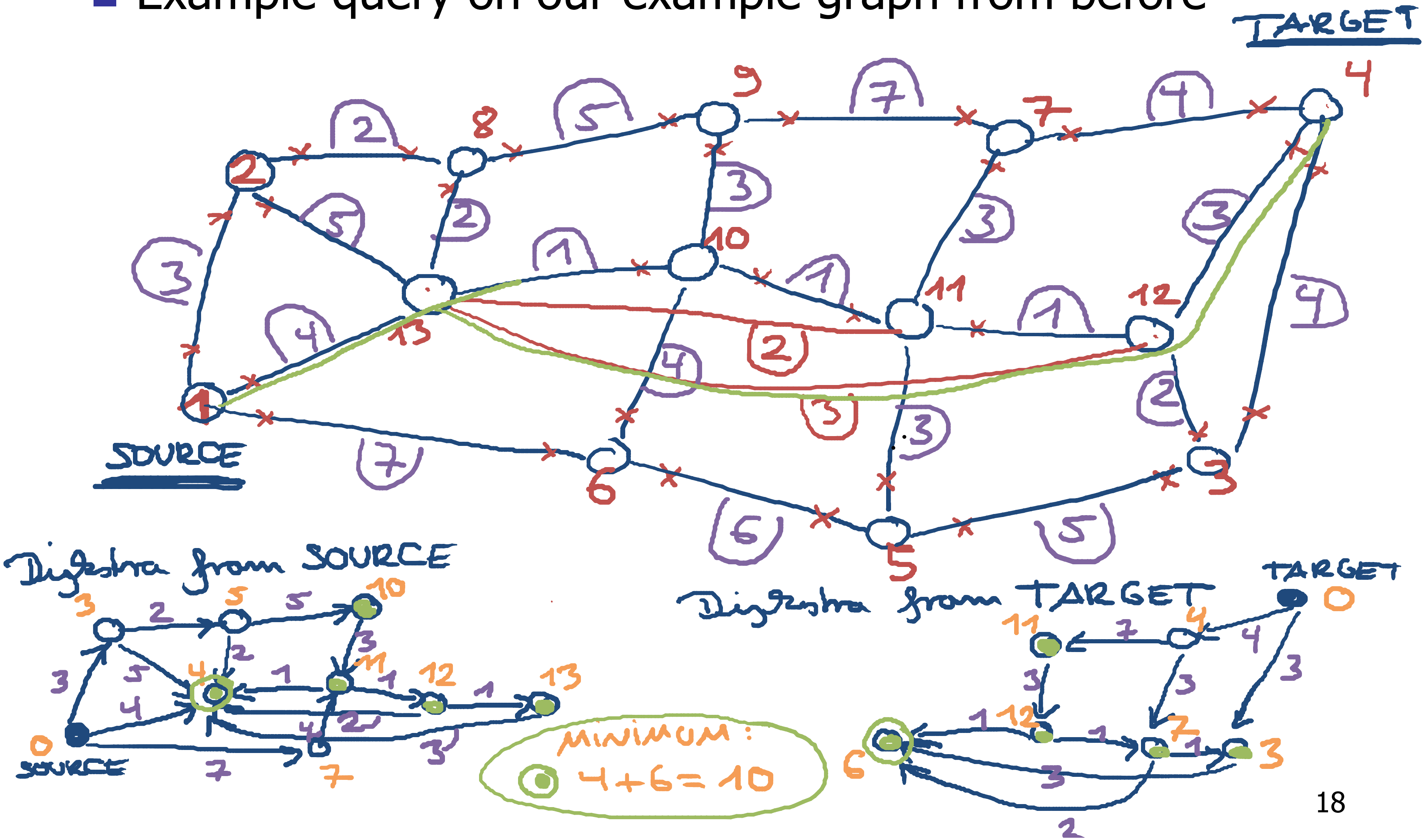
if • would have been contracted first, we would have had to add 6 shortcuts only for this contraction!

CH — Query algorithm 1/2

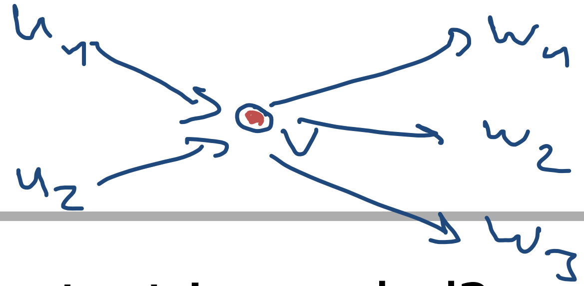
- Given $G^* = (V, E^*)$ and a source s and a target t
  - Define the upwards graph $G^*\uparrow = (V, \{(u, v) \in E^* : v > u\})$
  - Define the downwards graph $G^*\downarrow = (V, \{(u, v) \in E^* : v < u\})$
  - Do a full Dijkstra computation from s **forwards** in $G^*\uparrow$
  - Do a full Dijkstra computation from t **backwards** in $G^*\downarrow$
  - Let I be the set of nodes settled in **both** Dijkstras
  - Take $\text{dist}(s, t) = \min \{\text{dist}(s, v) + \text{dist}(v, t) : v \in I\}$
  - Is this correct and if yes why? ... next lecture
  - In the implementation, we need not construct $G^*\uparrow$ and $G^*\downarrow$ explicitly, we can just work on G^* ... next lecture
  - In symm. graphs backw. on $G^*\downarrow$ = forw. on $G^*\uparrow$ ... next lecture

CH — Query algorithm 2/2

- Example query on our example graph from before

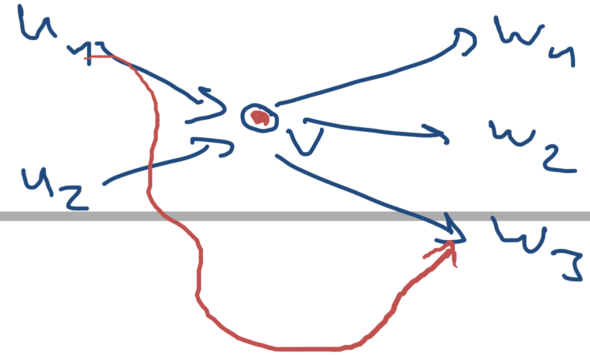


Shortcuts 1/3



- How to determine when a shortcut is needed?
  - Recall: when contracting node v, we need to insert the shortcut arc (u, w), if and only if $(u, v) \in E$ and $(v, w) \in E$ and (u, v, w) is the only shortest path from u to w
  - As before, $\{u_i\}$ = incoming arcs and $\{w_j\}$ = outgoing arcs
  - Perform a Dijkstra **for each u_i** in the graph **without v**
  - Let $D_{ij} = cost(u_i, v) + cost(v, w_j)$ ... cost of path via v
  - In the Dijkstra from u_i
    - ... stop when node with cost $> \max_j D_{ij}$ is settled
    - ... add shortcut (u_i, w_j) if and only if $dist[w_j] > D_{ij}$

Shortcuts 2/3



- Correctness of this routine
  - Assume there is a SP from u_i to w_j that does **not** pass through v
    - then the cost of that SP is $\leq D_{ij}$ and the Dijkstra from u_i just described will not stop before it has found it
    - then $dist[w_j] \leq D_{ij}$ and indeed no shortcut is added
  - Beware: there might be a SP through v with cost $< D_{ij}$
    - that looks like a problem, because this might be shorter than the SP in the graph without v
    - and we might not add a shortcut although we should
    - But such a path will then contain $(u_{i'}, v, w_{j'})$
    - And this will be taken care of by the Dijkstra from $u_{i'}$

Shortcuts 3/3

- Heuristic improvement
  - For each Dijkstra computation (from each of the u_i), put a **limit** on the size of the search space (#nodes settled)
    - With this heuristic, we may fail to find a shortest path from u_i to w_j that does not use v, and thus insert the shortcut (u_i, w_j) **unnecessarily**
    - But unnecessary shortcuts do not harm correctness, only performance (if there are too many of them)
    - So there is a trade-off: if the heuristic saves a lot of time in the precomputation at the cost of only a few unnecessary shortcuts, than it is worth it
  - Various additional heuristics in the paper ... see references

Implementation advice 1/2

- How to add shortcuts / remove contracted nodes?
  - If you implemented the adjacency lists with an Array<Array<Arc>>, adding arcs is straightforward
  - But make sure that either your Dijkstra implementation does not have a problem with the same arc existing twice ... or that you avoid adding an already existing arc
  - Removing nodes / arcs from the graph is more cumbersome, but luckily there is **no need** to do that
  - Instead, you can just ignore the respective nodes / arcs
  - In the precomputation, when contracting u_i, simply **ignore** all nodes $u_1, \ldots u_{i-1}$ and their adjacent arcs
  - You can use Arc::arcFlag for that ... see code suggestion

Implementation advice 2/2

- The Dijkstra searches for each contraction
  - ... should take only very little time (<< 1 millisecond)
    - for the full CH algorithm, we have to do one per node
  - To achieve that, pay attention to the following
    - make sure that the Dijkstra search spaces are small ... see the three slides on "Shortcuts"
    - requires two trivial extensions of DijsktrasAlgorithm class ... see code design suggestion linked on Wiki
    - avoid resetting the dist value for **every** node ... this would take $\Theta(n)$ time for each (tiny) Dijkstra
    - instead only reset the dist values of nodes that were visited in the previous Dijkstra (visitedNodes array)

References

- Highway Hierarchies

  Engineering Highway Hierarchies

  Highway Hierarchies Hasten Exact Shortest Path Queries

  Dominik Schultes and Peter Sanders, ESA 2005 & 2006

  http://algo2.iti.uka.de/schultes/hwy/esa06HwyHierarchies.pdf

  http://algo2.iti.uka.de/schultes/hwy/esaHwyHierarchies.pdf

- Contraction Hierarchies

  Contraction Hierarchies: Faster and Simpler Hierarchical Routing in Road Networks

  Geisberger, Sanders, Schultes, Delling, WEA 2008

  http://algo2.iti.uka.de/schultes/hwy/contract.pdf